Black Holes

Escape Speed

- The speed required to escape the gravitational attraction of a massive object is:

\[ v_{\text{esc}} = \frac{2GM}{R} \]

For the Earth, the escape speed is 11 km/s (25,000 miles per hour!). The radius of the Earth is 6400 km.

If we compressed the Earth to a radius of 1 cm, what would the escape speed be?

Set it up as a ratio using \( v_{\text{esc}} \propto \sqrt{1/R} \) since the mass is staying the same:

\[
v_{\text{esc,new}} = v_{\text{esc,old}} \sqrt{\frac{R_{\text{old}}}{R_{\text{new}}}} = (11 \text{ km/s}) \sqrt{\frac{6400 \times 10^5 \text{ cm}}{1 \text{ cm}}} \approx 2.8 \times 10^5 \text{ km/s}
\]

- The speed of light is \( 3 \times 10^8 \) m/s. How does this speed compare to the escape speed of a 1 cm radius Earth? What have we made?

This escape speed almost equals the speed of light, so we’ve just about made a black hole!

- If we compressed the Sun to a small enough volume, it would be a black hole. Would the Sun’s gravitational pull on the Earth change if the Sun were a a black hole? (Hint: Write down Newton’s Law of Universal Gravity, and see what changes.)

No - the gravitational pull would stay the same. Since \( F_g = GMm/r^2 \) and the mass isn’t changing and the distance between the Earth and the Sun/black hole stays the same, the force of gravity exerted by the black hole is the same as the Sun.

- Could the Earth still orbit the Sun, or would it be sucked in?

Since the gravitational pull stays the same (and the gravitational pull defines the orbit), the Earth would keep orbiting as it was before - it would not get sucked in.

Gravitational Redshift

- If you toss a ball up in the air, the Earth’s gravity pulls it downward. What happens to the kinetic energy of the ball as it is still moving upward? Energy cannot be created or destroyed, so where did the kinetic energy go?

The kinetic energy goes into moving further away from the Earth (it goes into potential energy).

- Say we stand on the surface of a neutron star (somehow!) and shine a green laser upwards. General relativity says that light always travels at the same speed: \( 3 \times 10^8 \) m/s. Can the green photons from the laser lose energy somehow while still traveling at the speed of light?

The energy of a photon depends on its wavelength: \( E = hc/\lambda \). So since a photon can’t slow down as it moves away from the neutron star, it must lose some of it’s intrinsic energy and become a lower energy photon: ie. the wavelength of the photon must increase. The photon is redshifted!
Time Dilation

- Lets consider a really weird clock, a clock that ticks every time the photons from a laser beam have a trough. ¹ To help visualize this sort of clock, sketch a light wave and write tick next to each trough.

- Now, lets send our laser clock down towards the outskirts of a black hole (outside the event horizon somewhere, so the light can get out). Since black holes are dangerous, well send the weirdo assigned roommate from our freshman year in the dorms to hold the laser, and well watch through a big telescope from a safe distance far away. As the ex-roommate gets closer to the event horizon, what happens to the energy of the laser light photons?
  The energy gets smaller and smaller as the photons are being emitted closer and closer to the black hole.

- So what happens to the wavelength of the photons?
  The wavelength of the photons gets longer and longer.

- If we measure time based on this laser clock, what do we see about the speed of the ticking laser clock?
  It slows down.

- Right at the event horizon, general relativity says that the photon must use all of its energy to get out of the gravitational pull of the black hole. What happens to the wavelength of the laser clock photons when the clock reaches the event horizon?
  The wavelength gets infinitely long.

- What can you say about how time passes at the event horizon as seen far away?
  Time seems to stop!

- Will our (doomed!) ex-roommate notice this weird time dilation as she falls into the black hole? (The laser clock is right there in her hand!) What would she say about your clock?
  No - time will seem to pass normally to her. She will think your clock is actually ticking faster and faster since as the photons from your laser fall into the black hole, they are blueshifted.

¹Actually, its not that weird - the official definition of one second of time is 9,192,631,770 cycles of microwave light emitted by a cesium atom! This is called an atomic clock, and unlike a grandfather clock, it never needs to be wound.