One square meter of Earth’s surface receives 1370 watts of power (1370 joules per second, or with the units from class, $1.37 \times 10^{10}$ ergs per second) of solar energy - enough to power about 20 light bulbs.

1. Imagine that we build a gigantic hollow shell completely encasing the Sun just outside Earth’s orbit. What would its surface area be (in square meters)\(?\) (SA = $4\pi r^2$, 1 AU = $1.5 \times 10^{11}$ m)

$$SA = 4\pi (1.5 \times 10^{11} \text{ m})^2 \approx 2.8 \times 10^{23} \text{ m}^2$$

2. What total power (in watts) would the entire mega-shell receive from the Sun?

The total power received by the entire mega-shell is just the area of the shell times the amount of power received by one square meter of the Earth’s surface (1370 watts):

$$Total\ Power = (2.8 \times 10^{23} \text{ m}^2) \times (1370\ watts\ per\ m^2) \approx 3.9 \times 10^{26}\ watts$$

3. Use your answer from part 2 to write down the luminosity (total power emitted) of the Sun in ergs per second (1 watt = $10^7$ erg/s).

The luminosity of the Sun is just the total power we calculated in part 2, $3.9 \times 10^{26}$ watts. To convert this to erg/s, we just multiply by $10^7$ to get that the luminosity = $3.9 \times 10^{33}$ erg/s.

4. The largest thermonuclear bomb ever exploded released about 250,000 terajoules ($2.5 \times 10^{17}$ joules) of total energy. How many thermonuclear bombs per second would have to be exploded to generate the amount of power released by the Sun?

The Sun puts out $3.9 \times 10^{26}$ Joules every second, so that means that you would need $3.9 \times 10^{26}/2.5 \times 10^{17} \approx 1.6 \times 10^9$, or about 1.6 billion thermonuclear bombs to go off every second in order to equal the power emitted by the Sun!
How Do Quantities Scale?

\[ \text{Brightness} = \frac{L}{4\pi d^2} \quad L = 4\pi R^2 \sigma T^4 \]

1. If stars A and B have the same luminosity, but star A is twice as far away as star B, how much brighter or dimmer would star A appear (compared to the brightness of star B)?

To solve this ratio problem, we start by comparing the brightness equations for stars A and B:

\[ \frac{b_A}{b_B} = \frac{L_A}{4\pi d_A^2} \frac{d_B^2}{L_B} \left( \frac{d_B}{d_A} \right)^2 \]

The problem states that the stars have the same luminosity \((L_A = L_B)\) and star A is twice as far away, so \(d_A = 2d_B\). Plugging this in,

\[ \frac{b_A}{b_B} = (1) \left( \frac{d_B}{2d_B} \right)^2 = \left( \frac{1}{2} \right)^2 = \frac{1}{4} \]

Therefore, star A is 1/4 as bright as star B, or star A is 4 times dimmer than star B.

2. Now let’s say that star A appears 8 times brighter than star B. We know that stars A and B are the same distance from us, so what does this tell us about their luminosities?

We do the same thing as above, but plugging in \(b_A = 8b_B\) and \(d_A = d_B\):

\[ 8 = \frac{L_A}{L_B} \left( \frac{d_B}{d_A} \right)^2 = \frac{L_A}{L_B} (1)^2 = \frac{L_A}{L_B} \]

So the luminosity of star A is 8 times the luminosity of star B.

3. What is the ratio of the luminosities of stars A and B \((L_A/L_B)\) if stars A and B have the same temperatures but the radius of star A is 3 times larger than the radius of star B?

Now we are talking about luminosity, radius and temperature, so we use \(L = 4\pi R^2 \sigma T^4\). Doing the same trick of dividing this equation for star A by the equation for star B, we find:

\[ \frac{L_A}{L_B} = \frac{4\pi R_A^2 \sigma T_A^4}{4\pi R_B^2 \sigma T_B^4} = \left( \frac{R_A}{R_B} \right)^2 \left( \frac{T_A}{T_B} \right)^4 \]

So plugging in \(T_A = T_B\) and \(R_A = 3R_B\), we find \(L_A/L_B = 9\).

4. What is the ratio of the luminosities \((L_A/L_B)\) if the temperature of star A is twice the temperature of star B but the radius of star A is 1/4 the radius of star B?

Same as above, but now \(T_A = 2T_B\) and \(R_A = 1/4R_B\):

\[ \frac{L_A}{L_B} = \left( \frac{R_A}{R_B} \right)^2 \left( \frac{T_A}{T_B} \right)^4 = \left( \frac{1}{4} \right)^2 (2)^4 = \frac{64}{16} = 4 \]
5. If the luminosity of star A is 64 times that of star B and we know that the temperature of star A is twice that of star B, what is the ratio of their radii ($R_A/R_B$)?

Same as above, but now $L_A = 64 L_B$ and $T_A = 2 T_B$:

$$64 = \left( \frac{R_A}{R_B} \right)^2 \left( \frac{2}{1} \right)^4 = 16 \left( \frac{R_A}{R_B} \right)^2$$

Therefore,

$$\frac{R_A}{R_B} = \sqrt{\frac{64}{16}} = \sqrt{4} = 2$$