Equations you need to know in full:

- **Doppler Shift:**
  \[
  \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta \lambda}{\lambda_0} = \frac{v}{c}
  \]
  - \(\lambda\) is the *observed* wavelength
  - \(\lambda_0\) is the *emitted* wavelength or the wavelength *at rest* that you would measure in the lab
  - \(v\) is the *relative* velocity *along the line of sight*: positive \(v\) means the object is moving away from you (redshift) and negative \(v\) means the object is moving towards you (blueshift)
  - \(c\) is the speed of light \((3 \times 10^{10} \text{ cm/s})\).

  When solving a Doppler shift problem, be careful about deciding which given wavelength is \(\lambda\) and which is \(\lambda_0\). Also be mindful of the fact that \(v\) is along the line of sight: recall the orbiting satellite problem in which the satellite’s velocity was *perpendicular* to the direction in which the light was emitted (the line of sight) so that there was no component of the satellite’s velocity along the line of sight and therefore, *no Doppler shift*!

- **Parallax:**
  \[
  d(\text{in pc}) = \frac{1}{p(\text{in arcseconds})}
  \]
  When calculating the distance to a nearby star using parallax, you must plug in the angle of parallax \(p\) in *arcseconds* and the distance \(d\) in *parsecs*! The formula will give incorrect answers if you use other units. Make sure you understand how parallax works (see pages 249-250 in the textbook).

Equations for which you just need to know the scaling relations:
For the following equations, you do not need to memorize the full form of the equations, just the scaling relations.

- **Wien’s Law**
  \[
  \lambda_{peak} \cdot T = 0.29 \text{ cm} \cdot \text{K} \Rightarrow \lambda_{peak} \cdot T = \text{constant} \quad \text{or} \quad \lambda_{peak} \propto \frac{1}{T}
  \]
  - \(\lambda_{peak}\) is the wavelength at which the blackbody spectrum peaks: the wavelength at which the blackbody emits most of its radiation
  - \(T\) is the temperature of the blackbody in general and in the case of stars, it is the temperature of the photosphere of the star.

  This is simply a property of blackbodies. We apply it to stars since stars behave like blackbodies.
Kepler’s Third Law (Newton’s form)

\[ P^2 = \frac{4\pi^2}{G(M + m)a^3} \Rightarrow P^2 \propto \frac{a^3}{(M + m)} \]

- \( P \) is the period of the orbit
- \( G \) is the constant of gravitation, \( 6.67 \times 10^{-8} \) dyne cm\(^2\) g\(^{-2}\)
- \( a \) is the semi-major axis of the orbit, or the radius of the orbit for circular orbits (don’t confuse this with acceleration!)
- \( M \) is the mass of the larger or central object
- \( m \) is the mass of the smaller object (the orbit of which is described by \( P \) and \( a \))

For problems involving a moon orbiting a planet or a planet orbiting a star, the smaller mass is much smaller than the larger mass, so we can ignore it (i.e. \( M + m \approx M \)). Therefore, the scaling relation just reduces to:

\[ P^2 \propto \frac{a^3}{M} \]

where \( M \) is the larger mass (the central object).

Luminosity of Stars

\[ L = 4\pi R^2 \sigma T^4 \Rightarrow L \propto R^2 T^4 \]

- \( L \) is the luminosity of the star
- \( R \) is the radius of the star
- \( \sigma \) is the Stefan-Boltzmann constant, \( 5.67 \times 10^{-5} \) erg cm\(^{-2}\) s\(^{-1}\) K\(^{-4}\)
- \( T \) is the temperature of the photosphere of the star

Recall that \( \sigma T^4 \) is the energy per second output by 1 cm\(^2\) of a blackbody, so the total energy per second output (the luminosity) is just this multiplied by the surface area of the star, \( 4\pi R^2 \).

Brightness

\[ b = \frac{L}{4\pi d^2} \Rightarrow b \propto \frac{L}{d^2} \]

- \( b \) is the observed brightness, which is energy per second per cm\(^2\)
- \( L \) is the luminosity (energy per second)
- \( d \) is the distance between the observer and the source with luminosity \( L \).

Recall that luminosity is the total amount of energy per second output by a star, but brightness is how much energy per second is received by a 1 cm\(^2\) receiver (like your eye) a distance \( d \) away from the star.
Example Problems

1. If Star A’s luminosity is 4 times that of star B and star A’s temperature is twice that of star B, what is the ratio of their radii?

2. Saturn orbits approximately 10 AU from the Sun. How much brighter does the Sun appear to us on Earth than it would appear to an alien living on Saturn.

3. A planet orbiting a distant star has a period of 4 years. The mass of the star is half the mass of the Sun. What is the semi-major axis of this planet’s orbit?

4. Looking for exoplanets, we observe a star’s spectral line oscillating between 299.95 nm and 300.05 nm. This particular spectral line has a wavelength of 300 nm at rest (in the lab). What is the orbital velocity of the star (i.e., its velocity as it wobbles due to the presence of the planet)?

5. Star A is 3 times as hot as star B. Star B emits most of its radiation at 600 nm. At what wavelength does star A emit most of its radiation?

6. A star is observed to have a parallax of 0.05 arcseconds. How far away is it?

7. If a comet comes around about every 8 years, what is the semi-major axis of its orbit?
Two methods for setting up a scaling relation problem (aka the Ratio Method!):

- One method for setting up one of these problems is to divide the formula for one case by the formula for the other case. For example, take the scaling relation for brightness: \( b \propto L/d^2 \) where \( L \) is the luminosity of the star and \( d \) is the distance between the star and the observer. Suppose we want to find the ratio of the brightnesses of stars 1 and 2, i.e., we want to find \( b_1/b_2 \). Set it up like this:

\[
\frac{b_1}{b_2} = \frac{\frac{L_1}{d_1^2}}{\frac{L_2}{d_2^2}} = \left( \frac{L_1}{L_2} \right) \left( \frac{d_2}{d_1} \right)^2
\]

Now the question might say that the luminosity of star 1 is 3 times that of star 2 and that the distance to star 1 is half the distance to star 2. This means that \( L_1 = 3L_2 \) and \( d_1 = \frac{1}{2}d_2 \). So we can just plug these into the formula above:

\[
\frac{b_1}{b_2} = \left( \frac{3L_2}{L_2} \right) \left( \frac{d_2}{(1/2)d_2} \right)^2 = (3)(2)^2 = 3 \cdot 4 = 12
\]

So \( b_1/b_2 = 12 \) which means that star 1 appears 12 times brighter than star 2.

- Another method for setting up scaling relation problems is to first rearrange the scaling relation so that all the variables are on one side of the equation (so that the other side is just a constant). For example, the brightness equation above, \( b \propto L/d^2 \), becomes \( bd^2/L \propto 1 \) which just means that \( bd^2/L = \text{constant} \). Since \( bd^2/L \) is always equal to the same constant for any case, you can plug in the values for star 1 and set that equal to values for star 2:

\[
\frac{b_1 d_1^2}{L_1} = \frac{b_2 d_2^2}{L_2}
\]

With a little algebra, you can rearrange this formula so solve for whatever you want. Let’s say that the question asks you to solve for the ratio of the distances \( (d_1/d_2) \) given that star 1 appears 12 times brighter than star 2 (so \( b_1 = 12b_2 \)) and that the luminosity of star 1 is 3 times that of star 2 (so \( L_1 = 3L_2 \)). Doing a little algebra, we rearrange the equation to solve for \( d_1/d_2 \) and plug in the given values:

\[
\frac{d_1}{d_2} = \sqrt{\frac{b_2}{b_1} \frac{L_1}{L_2}} = \sqrt{\frac{b_2}{b_1} \frac{3L_2}{L_1}} = \sqrt{\frac{3}{12}} = \sqrt{\frac{1}{4}} = \frac{1}{2}
\]

Therefore, \( d_1 \) is just 1/2 of \( d_2 \): star 1 is only half as far away as star 2.

**NOTE:** Some problems may not give you two cases to compare. If this happens, think about setting up the ratio with a case you know (like the Earth’s orbit around the Sun!)

**Answers to example problems:**

1. \( R_A/R_B = 1/2 \)
2. 100 times brighter
3. 2 AU
4. \( 5 \times 10^6 \) cm/s
5. 200 nm
6. 20 parsecs
7. 4 AU