Astronomy 84

Questions for March 5

1. Try re-deriving the degenerate Fermi gas equation of state that’s don on page 144 of your text, but now simply starting from confining each electron to a box of dimension $\Delta x = \left(\frac{2}{n_e}\right)^{1/3}$ instead of confining them to the whole star. Why choose this box size as input to the uncertainty principle? Why the factor 2? Compare your result to Schutz’s expression 12.13, and comment on whether he or you have to assume $n_e$ to be independent of position in the star.

2. The following figure shows the binding energy of atomic nuclei, as a function of the total number of nucleons in a nucleus.

There’s a gentle maximum at $^{56}\text{Fe}$. a) Qualitatively, why is there a maximum? Hint: the protons are all positively charged, thus repel each other
electrostatically – that force is long range, with the repulsion between two protons declining in proportion to (particle separation)^2; the strong nuclear force holds the nucleus together, but drops to zero for separations larger than one or two Fermi (1 fermi = 10^{-15} m). Heavy nuclei such as Uranium have more neutrons than protons so as to be stable – why? b) Massive stars restore the energy radiated by nuclear “cooking”, converting lighter nuclei to heavier. Is there such net energy release obtained by cooking nuclei to elements heavier than iron? c) In the cores of massive stars, contraction of the core raises the temperature, so as to keep the cooking going (see text). When hot enough, the photons have enough energy to “photo-evaporate” the heavy nuclei, by colliding with the nuclei and transferring energy in excess of the binding energy. Does the store of trapped thermal energy have to pay the price of such evaporation? If it does, what happens to the star’s stability (refer to the text’s discussion of why a star is stable, and think about the effects of a compression, if the increased energy can go to photoevaporation rather than to increasing the pressure.) Does the core where nuclear reactions occur collapse.