Lec. 4 Thermal Properties & Line Diagnostics for HII Regions

1. General Introduction*
2. Temperature of Photoionized Gas: Heating & Cooling of HII Regions
3. Thermal Balance
4. Line Emission
5. Diagnostics

*Heating and cooling for the ISM, not just HII regions.

References: Spitzer, Chs. 5-6 & Osterbrock, Ch3. 3-5
Introduction to Thermal Properties

The basic hydrodynamic equations state the conservation laws for continuum problems.

After mass and momentum conservation, the entropy equation, known as the heat equation, is the most important.

The volumetric rates of gain and loss of entropy per unit volume are called the heating rate $\Gamma$ and the cooling rate $\Lambda$.

In steady state, these two rates must balance. Solving $\Gamma = \Lambda$ yields the temperature $T$. 
Line Cooling Function

\( \Lambda \) is the net rate at which energy is lost in collisions. For HII regions, electrons are the most important collision partners. The collisions are described by upward and downward rate coefficients \( k_{lu} \) & \( k_{ul} \) related by detailed balance.

NB Instead of \( j \) and \( k \) for lower and upper level, we use \( u \) and \( l \)
The Escape Probability

By analyzing the steady rate equations for the level population and introducing the escape probability, the cooling can be expressed in a compact form that resembles the emissivity of the transition.

Statistical Equilibrium

Assume that external (or ambient) radiation field is weak.

The population of the levels is determined by collisions and spontaneous decay. For level \( m_u \), balance excitation rate against de-excitation rate:

\[
\sum \frac{m_u C_{m_u}}{Z} = \sum \frac{m_u C_{m_u}}{Z} + \sum \frac{\beta_{m_u}}{\nu_s} A_{m_u} M_{m_u},
\]

where we have included an escape probability to take account of absorption by the source. Usually it has the form

\[
\beta_{m_u} = \frac{1 - e^{-\nu_s}}{\nu_s},
\]

where the exact form of \( \nu_s \) varies from problem to problem. Also, \( C_{m_u} = m_u k_{m_u} \).

Substituting into \( \Lambda \) (previous page) leads to

\[
\Lambda = \sum \beta_{m_u} A_{m_u} E_{m_u} M_{m_u}.
\]
Model Two-Level System

Exactly solvable using escape probability.

The closed form of the population of the upper level leads to a simple cooling function that involves the critical density:

\[ n_{cr} = \frac{A_{ul}}{k_{ul}} \]

High densities \((n >> n_{cr})\) give the TE result.

Low densities \((n << n_{cr})\) give a quadratic density cooling function.
2. Temperature of Photoionized Gas

What are the heating and cooling processes that determine the thermal balance $\Gamma = \Lambda$ for photoionized regions?

There is essentially only one heating process when UV photons from hot stars are absorbed, the energy dissipated by the photoelectrons in Coulomb collisions.

$$E_e = h\nu - h\nu_1 \sim kT$$

The mean energy of the photoelectrons is

$$\overline{E}_2 = \frac{\int_{\nu_1}^{\infty} h(\nu - \nu_1)\sigma_\nu \frac{4\pi J_\nu}{h\nu} d\nu}{\int_{\nu_1}^{\infty} \sigma_\nu \frac{4\pi J_\nu}{h\nu} d\nu}$$

where $J_\nu$ is the mean intensity of the radiation field.
Photoelectric Heating

Following Spitzer Sec. 6.1, express the mean photoelectron energy in terms of the stellar effective temperature

\[ \psi = \frac{\bar{E}_2}{kT_*} \]

With \( \zeta \pi n_H \) the photoionization rate per unit volume, the volumetric heating rate is

\[ \Gamma = \zeta \pi n_H \psi kT^* \]

<table>
<thead>
<tr>
<th>( T_e(\text{K}) )</th>
<th>4000</th>
<th>8000</th>
<th>16,000</th>
<th>32,000</th>
<th>64,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi_0 )</td>
<td>0.977</td>
<td>0.959</td>
<td>0.922</td>
<td>0.864</td>
<td>0.775</td>
</tr>
<tr>
<td>( \langle \psi_0 \rangle )</td>
<td>1.051</td>
<td>1.101</td>
<td>1.199</td>
<td>1.380</td>
<td>1.655</td>
</tr>
</tbody>
</table>

\( \psi_0 \) - value near star, \( \langle \psi_0 \rangle \) is averaged over an HII region

Spitzer, Table 6.1
Comment on Photoelectric Heating

The spectrum hardens because of $\nu^{-3}$ absorption

The mean photoelectron energy increases, $\psi_0 > \langle \psi_0 \rangle$, as shown in the previous table.
Comment on Recombination Cooling

Every recombination drains thermal energy $\frac{1}{2} m_e w^2$ from the gas. The total cooling rate per ion is

$$\frac{1}{2} m \sum_{j=k}^{\infty} \langle w^3 \sigma_j \rangle$$

Recall that the recombination cross section varies as the inverse square of the speed $w$. The recombination rate

$$\langle \sigma_i w \rangle \sim \left\langle \frac{g_{nf}}{w^2} w \right\rangle \sim \left\langle \frac{g_{nf}}{w} \right\rangle$$

favors slow electrons and varies as $T^{-1/2}$. 
Recombination Cooling: Conclusion

The volumetric cooling rate (on-the-spot approximation, neglecting recombinations to the ground state) is

\[ \Gamma_{\text{rec}} = \alpha n_e n_H kT \left( \frac{\chi}{\phi} \right) \]

\(\chi\) energy gain function, \(\phi\) recombination function, c.f. Spitzer Sec 6.1; see his Eq. (609) below.

<table>
<thead>
<tr>
<th>T/K</th>
<th>4000</th>
<th>8000</th>
<th>16,000</th>
<th>64,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\chi^{(2)}/\phi^{(2)})</td>
<td>0.73</td>
<td>0.69</td>
<td>0.63</td>
<td>0.51</td>
</tr>
</tbody>
</table>

\(\approx 2/3 \ kT\) electron energy is lost per recombination

\[ \Gamma_{\text{ep}} = \frac{2.07 \times 10^{-11} n_e n_p}{T^{1/2}} \left\{ \overline{E_2 \Phi_1(\beta)} - kT \chi_1(\beta) \right\} \frac{\text{erg}}{\text{cm}^3 \text{s}}, \quad (6-9) \]
3. Thermal Balance for Pure H

Spitzer subtracts recombination cooling from photo-electric heating:

\[
\Gamma_{ep} = \frac{2.07 \times 10^{-11} n_e n_p}{T^{1/2}} \left\{ \bar{E}_2 \phi_1(\beta) - kT \chi_1(\beta) \right\} \frac{\text{erg}}{\text{cm}^3 \text{s}},
\]

Therefore the standard thermal balance condition \( \Gamma = \Lambda \) is now \( \Gamma_{ep} = 0 \), or

\[
\frac{T}{T^*} = \left( \chi/\phi \right) \langle \psi_0 \rangle \sim 1.5 \langle \psi_0 \rangle
\]

For \( T^* = 32,000 \text{K (B0.5)} \), \( \langle \psi \rangle = 1.38 \), \( T = 2.1 \), \( T^* = 66,000 \text{K} \), much hotter than observed. There must be other coolants (not surprising since we observe many emission lines).
4. Line Emission

Balancing photoelectric heating and recombination heating in a pure hydrogen model predicts too high temperatures for HII regions.

The observed optical line emission from HII regions is dominated by the recombination lines of H & He and by the forbidden lines of heavy elements. What is missing is cooling by line emission.

Two improvements are needed in the photoionization model: inclusion of heavy elements & collisional excitation.

Reference: The exhaustive treatment in Osterbrock, Chs. 4 & 5.
Long Slit Optical Spectrum of the Orion Bar Region

Heavy element forbidden transitions are stronger than the hydrogen lines, especially OIII.

Even more so for SNRs and AGN in this spectral region.
Grotrian Diagram for OIII

Ground configuration (from the bottom of the diagram, not to scale)

\[ 1S_0 \quad \overset{4363}{\text{------------------------}} \]
\[ 1D_2 \quad \overset{4959, 5007}{\text{------------------------}} \]
\[ 3P_J \quad \overset{\text{Fine structure of ground level not shown}}{\text{------------------------}} \]

Why are these transitions called forbidden lines?
Historical Note on Nebular Lines

**Helium** – discovered in 1868 by Janssen in solar chromosphere (in eclipse) at 5816 Angstroms.

Identified as non-terrestrial by Lockyer & Franklund; later detected in minerals.

*Significance:* 40% of the visible mass had been missed (although the fact that most of it is hydrogen was unknown then).

**Nebulium** – discovered by Huggins in 1864 in nebulae at 5007, 4959 and 3726, 3729 Angstroms; as for He, ascribed to a new non-terrestrial element.

Identified in 1927 by Bowen as OIII and OII.

*Significance:* highlighted the possibility of long-lived quantum states and focused attention on understanding *selection rules* in quantum mechanics.
5. Diagnostics of Physical Conditions

Example: ground configuration energy levels for OIII & NII

For more precise wavelengths, A-values, see Osterbrock Table 3.8
Critical Densities

The cooling depends critically on how the electron density compares with the critical density

\[ n_{cr} = \frac{A_{ul}}{k_{ul}}, \]

The collisional rate coefficients for e-ion collisions is given by a standard form (Osterbrock Sec. 3.5)

\[ k_{ue} = \frac{8.6 \times 10^{-6}}{T^{\frac{3}{2}}} \frac{\omega_{e}}{\nu_{e}} \]

\[ \omega_{e} = \text{collision strength (O4)} \]

\[ k_{ue} = \frac{\nu_{e}}{\nu_{*}} e^{-\frac{E_{ue}}{k_{B}T}} k_{ue} \text{ detailed balance} \]

Typical values of \( k \) are \( 10^{-8} \) for downward and \( 10^{-10} \) for upward rate coefficients.
Solution of the Temperature Problem for HII Regions

Recall that Spitzer’s $\Gamma_{ep}$ (dashed lines) has the recombination cooling subtracted from the photoelectric heating.

The solid lines are line cooling plus a small free-free component. NII & OIII are the most important.
How to Measure $T$ and $n_e$ For HII Regions

*With spectroscopy, of course!*

**Measuring Temperature:** The mere occurrence of optical forbidden lines suggests values of order $10^4$ K. (NB: $E/eV = 12,400$ ($Å/λ$) and 1eV corresponds to 11,605 K)

More precisely, observe levels of the same ion arising from different upper levels, e.g., ratio the intensities of the 5007/4959 and 4363 lines of OIII.

The ratio changes form 2000 to 400 as $T$ changes from 6,000 to 20,000 K
How to Measure $T$ and $n_e$ (cont'd)

Measuring Electron Density:

The critical densities vary from transition to transition because $A$-values and rate coefficients do, e.g., in the case of doublets leading to the ground state it is just from statistical weights.

For example, SII has strong red lines arising from the first excited level, a fine-structure doublet: $^2D_{5/2}, \ ^2D_{3/2} \rightarrow \ ^4S_{3/2}$ at 6716, 6731 Å. The critical densities are of order $10^4$ cm s$^{-1}$.

At low densities, the intensities are determined by the collision strengths, at high densities by the $A$-values. The net effect is a line-intensity ratio that varies significantly with density.
Combined Diagnostics Using OIII Optical and Far Infrared Lines

Osterbrock, Fig. 5.6

**Figure 5.6**
Calculated variation of [O III] forbidden-line relative-intensity ratios as functions of $T$ (9000° to 20,000° K) and $N_e$. Observed planetary-nebula ratios plotted with indication of probable errors.
Summary

We have discussed the processes that produce HII regions around young, massive stars and that generate diagnostic emission lines that can be used to measure T and \( n_e \).

The important forbidden lines of the heavy atoms & ions are excited by electronic collisions (in contrast to the recombination lines) so that, even in this simplest of photoionized nebulae, collisional phenomena play a role.

Similar methods apply to the study of planetary and AGN nebulae, although there are additional physical processes to consider, e.g., with regard to the difference between UV and X-ray ionization.

Completely missing from this discussion are dynamic effects, especially the fact that massive stars have powerful outflows.