Lecture 5. Interstellar Dust: Optical Properties

1. Introduction
2. Extinction
3. Mie Scattering
4. Dust to Gas Ratio
5. Appendices

References
Spitzer Ch. 7, Osterbrock Ch. 7
DC Whittet, Dust in the Galactic Environment (IoP, 2002)
E Krugel, Physics of Interstellar Dust (IoP, 2003)
B Draine, ARAA, 41, 241, 2003
1. Introduction: Brief History of Dust

Nebular gas long accepted but existence of absorbing interstellar dust controversial. Herschel (1738-1822) found few stars in some directions, later extensively demonstrated by Barnard’s photos of dark clouds.

Trumpler (PASP 42 214 1930) conclusively demonstrated interstellar absorption by comparing luminosity distances & angular diameter distances for open clusters:

• Angular diameter distances are systematically smaller
• Discrepancy grows with distance
• Distant clusters are redder
• Estimated ~ 2 mag/kpc absorption
• Attributed it to Rayleigh scattering by gas
Some of the Evidence for Interstellar Dust

Extinction (reddening of bright stars, dark clouds)
Polarization of starlight
Scattering (reflection nebulae)
Continuum IR emission
Depletion of refractory elements from the gas

Dust is also observed in the winds of AGB stars, SNRs, young stellar objects (YSOs), comets, interplanetary Dust particles (IDPs), and in external galaxies.

The extinction varies continuously with wavelength and requires macroscopic absorbers (or “dust” particles).
Examples of the Effects of Dust

- Extinction B68
- Scattering - Pleiades
Extinction: Some Definitions

Optical depth, cross section, & efficiency:

\[ \tau_{\lambda}^{\text{ext}} = \int n_{\text{dust}} \sigma_{\lambda}^{\text{ext}} \, ds = \sigma_{\lambda}^{\text{ext}} \int n_{\text{dust}} \]

\[ = \pi a^2 Q_{\text{ext}}(\lambda) N_{\text{dust}} \]

\( n_d \) is the volumetric dust density

The magnitude of the extinction \( A_\lambda \):

\[ I(\lambda) = I_0(\lambda) \exp[-\tau_{\lambda}^{\text{ext}}] \]

\[ A_\lambda = -2.5 \log_{10} \left[ I(\lambda)/I_0(\lambda) \right] \]

\[ = 2.5 \log_{10}(e) \tau_{\lambda}^{\text{ext}} = 1.086 \tau_{\lambda}^{\text{ext}} \]
2. Extinction (Continuum Opacity)

General Properties
1. uniform shape
2. $\lambda^{-1}$ trend in the optical/NIR
3. steep UV rise with peak at $\sim 800$ Å
4. spectral features:
   - $\lambda = 220$ nm $\Delta \lambda = 47$ nm
   - $\lambda = 9.7$ µm, $\Delta \lambda \sim 2-3$ µm
5. polarization with large extinction

![Magnitudes of extinction vs, wavelength (schematic)](image)
Extinction = Scattering + Absorption

For spherical grains, define efficiencies, $Q_{ext}$, $Q_{sca}$, $Q_{abs}$ such that

$$Q_{ext} = Q_{sca} + Q_{abs}$$

$$\sigma_{abs} = Q_{abs} \pi a^2$$

$$\sigma_{sca} = Q_{sca} \pi a^2$$

$$\sigma_{ext} = Q_{ext} \pi a^2 = (Q_{abs} + Q_{sca}) \pi a^2$$

albedo $= \frac{\sigma_{sca}}{\sigma_{ext}} = \frac{Q_{sca}}{Q_{ext}} \leq 1$
Scattering Phase Function

The scattered intensity is a function of the angle \( \theta \) between the incident and scattered wave, and is quantified by the *phase function* \( g \)

\[
g = \langle \cos \theta \rangle = \frac{\int_0^\pi I(\theta) \cos \theta \, d\Omega}{\int_0^\pi I(\theta) \, d\Omega}
\]

- Isotropic scattering \( \langle \cos \theta \rangle = 0 \)
- Forward scattering \( \langle \cos \theta \rangle = 1 \)
- Back scattering \( \langle \cos \theta \rangle = -1 \)
Measuring the Extinction

The absolute extinction requires the distance & absolute magnitude, \( m_\lambda = M_\lambda + 5 \log d - 5 + A_\lambda \); instead use:

The *relative extinction* or "selective extinction" is deduced from observations of reddened stars of known spectral type, i.e., starting from the known *color excess*, \((B-V)_0\)

\[
E(B-V) = A(B)-A(V) = (B-V) - (B-V)_0
\]

The *normalized extinction*, \(1/R_V\), measures the steepness of the extinction

\[
R_V = A(V) / [A(B)-A(V)] = A(V) / E(B-V)
\]

It is steep in the diffuse ISM: \(R_V = 3.1\pm0.2\), shallower in dark clouds: \(R_V \approx 5\)
Copernicus Observations of Two Similar Stars

Raw (uncorrected, unnormalized) count rates from the UV satellite for two O7 stars, S Mon (top, “un-reddened”) and ξ Per (bottom, “reddened”).

Upper panel: 1800 - 2000 Å

Bottom panel: 1000 – 2000 Å
Representative Interstellar Extinction Curves

Based on extinction measurements from bands from UV to NIR

Empirical Extinction to Gas Ratio

The extinction measurements for the diffuse interstellar medium yield a constant *extinction to gas ratio* for diffuse clouds near the Sun (with $R_V \approx 3.1$):

$$A (V) \approx N_H / 2 \times 10^{21} \text{ mag cm}^{-2}$$

This suggests that the *dust-to-gas mass ratio* is approximately constant since $A(V)$ is related to the column density of the absorbing dust:

$$A(V) = 1.086 \sum n_d \Delta s \sigma^{ext} = 1.086 N_d \sigma^{ext}$$

Dividing by the gas column, $N_H = \sum n_H \Delta s$, yields the mean extinction cross section per H nucleus

$$< n_d / n_H \sigma^{ext} > \approx 5 \times 10^{-21} \text{ cm}^2$$

See also Appendix 3.
3. Electromagnetic Scattering by Small Particles

AN Mie (Ann Phys 25 377 1908), considering uniform spheres of radius \( a \) with any index of refraction

\[ m = n - i k \quad \text{with} \quad m = m(\lambda), \]

used a *multipole expansion* of the scattered wave (“vector spherical harmonics” times radial Bessel functions) and applied boundary conditions at the surface of the sphere to get \( E \) and \( B \) for all space. Best reference is:

HC van de Hulst, “Light Scattering from Small Particles”

Basic parameter: \( x = \frac{2\pi a}{\lambda} \)

- \( x \ll 1 \): long wavelengths; need only a few terms
- \( x \gg 1 \): short wavelengths; need many terms
Asymptotic Formulae For $x << 1$

$$Q_{abs} = -4x \text{Im} \left( \frac{m^2 - 1}{m^2 + 2} \right) \propto \lambda^{-1}$$

$$Q_{sca} = \frac{8}{3} x^4 \text{Re} \left\{ \left( \frac{m^2 - 1}{m^2 + 2} \right)^2 \right\} \propto \lambda^4 \quad \text{(Rayleigh scattering)}$$

In this (long wavelength) limit, the absorption cross section depends only on the mass of the grain:

$$\sigma_{abs} = Q_{abs} \pi a^2 \propto a^3 \propto m_{dust}$$

We need to be able to calculate the optical properties of a wide variety of dust particle models in order to deduce the physical properties of IS dust from the observed extinction.
Pure vs. Dirty Dielectrics

$m = 1.33 + 0.00 \, i$

$m = 1.33 - 0.05 \, i$

$Q_{\text{ext}} = Q_{\text{sca}}$
Angular Distributions \((m = 1.33)\)

The angular distribution becomes highly peaked in the forward direction for \(x \gg 1\).

Note the difference between the E field perpendicular and parallel to the scattering plane.
4. Dust to Gas Ratio (Purcell 1969)

Use the Kramers-Kronig relation (ApJ 158 433 1960) to relate the integrated extinction coefficient for spherical grains with constant index $m$ to the dust column $N_d$

$$\int_0^\infty Q_{\text{ext}} \, d\lambda = 4 \pi^2 a \left( \frac{m^2 - 1}{m^2 + 2} \right)$$

$$\tau_{\text{ext}} = Q_{\text{ext}} \pi a^2 N_d, \quad N_d = \rho_d L / m_d$$

$$\int_0^\infty \tau_{\text{ext}} \, d\lambda = 4 \pi^3 a^3 N_d \left( \frac{m^2 - 1}{m^2 + 2} \right)$$

(only a lower limit on grain volume for other shapes)

Reintroduce the measured $A_{\lambda}$

$$A_{\lambda} = 1.086 \tau_{\lambda} = 1.086 Q_{\text{ext}} \pi a^2 N_d$$
Gas to Dust Ratio (cont’d)

\[ \int_{0}^{\infty} \frac{A_{\lambda} d\lambda}{L} = 3\pi^3 1.086 \left( \frac{m^2 - 1}{m^2 + 2} \right) n_d V_{gr} \]

\[ n_d V_{gr} = \frac{n_d m_{gr}}{m_{gr}/V_{gr}} = \frac{\rho_d}{\rho_{gr}} = \frac{\text{average dust density}}{\text{density of solid}} \]

where \( V_{gr} = (4\pi/3) a^3 \).

The average extinction \( A_{\lambda} \) (lower limit to integral) gives the mean dust density.

Dividing the integral by \( mn_H = mN_H/L \), gives the mean dust to gas mass ratio (\( m = \text{gas particle mass} \))

For example, assuming silicate dust with real index = 1.5 & \( \rho_{gr} = 2.5 \text{ g cm}^{-3} \) yields: \( \rho_d / \rho_g = 0.006 \).
Implications of Interstellar Abundances

With $\frac{\rho_d}{\rho_g} = 0.006$, a significant fraction of O and the refractories is locked up in interstellar dust, unless the present (compact, spherical) grain model is grossly incorrect.

For reference, the mass fraction of heavy elements $Z = 0.017$ (solar) and $Z = (B$ stars / ISM).

Many attempts have been made to make composition models within the abundance budget, e.g., typically
- Silicates: Mg, Si, Fe(95%), O (20%) in $(\text{Mg,Fe})_2\text{SiO}_4$
- Carbonaceous material (graphite & organics): C (60%)
- plus some SiC
## Exemplary Interstellar Dust Composition

<table>
<thead>
<tr>
<th>Element</th>
<th>Abundance</th>
<th>A</th>
<th>$M/M_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$0.6 \times 4.0 \times 10^{-4}$</td>
<td>12</td>
<td>0.0029</td>
</tr>
<tr>
<td>Mg</td>
<td>$4.0 \times 10^{-5}$</td>
<td>24</td>
<td>0.0010</td>
</tr>
<tr>
<td>Fe</td>
<td>$3.4 \times 10^{-5}$</td>
<td>56</td>
<td>0.0019</td>
</tr>
<tr>
<td>Si</td>
<td>$3.8 \times 10^{-5}$</td>
<td>28</td>
<td>0.0011</td>
</tr>
<tr>
<td>O</td>
<td>$0.2 \times 8.0 \times 10^{-4}$</td>
<td>16</td>
<td>0.0022</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>0.009</td>
</tr>
</tbody>
</table>

NB Both the abundances and the fractions of O and C used in this example are probably too high.
Another Interstellar Dust Composition

<table>
<thead>
<tr>
<th>Element</th>
<th>Abundance</th>
<th>A</th>
<th>$M/M_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.25 x 2.5x10^{-4}</td>
<td>12</td>
<td>0.0008</td>
</tr>
<tr>
<td>Mg</td>
<td>3.4x10^{-5}</td>
<td>24</td>
<td>0.0008</td>
</tr>
<tr>
<td>Fe</td>
<td>2.8x10^{-5}</td>
<td>56</td>
<td>0.0016</td>
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<tr>
<td>Si</td>
<td>3.4x10^{-5}</td>
<td>28</td>
<td>0.0010</td>
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<tr>
<td>O</td>
<td>0.28 x 4.9x10^{-4}</td>
<td>16</td>
<td>0.0022</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>0.0064</td>
</tr>
</tbody>
</table>

NB Here we’ve used the interstellar abundances from Lecture 01 and also adjusted the fractions of O and C, the latter from Draine’s 2003 review article.
Appendix 1.
Selective Extinction

DUST EXTINCTION
1. Selective Extinction

The extinction in magnitudes $A_{\lambda}$ is defined so that,

$$m_{\lambda} = m_{\lambda}^{(0)} + A_{\lambda},$$

(1)

where $m_{\lambda}$ and $m_{\lambda}^{(0)}$ are the magnitudes of two identical stars with and without dust. Usually $\lambda$ refers to broad band filters, e.g., $U$, $V$, $B$, etc.

The selective extinction is

$$E(\lambda_1, \lambda_2) = A_{\lambda_1} - A_{\lambda_2},$$

(2)

$E(\lambda_1, \lambda_2)$ is proportional to the amount of dust along the line of sight,

$$\tau_d(\lambda) = \int ds \sigma_d(\lambda) = \int ds n_H \left( \frac{n_d}{n_H} \sigma_d(\lambda) \right),$$

(3)

consistent with the earlier statement,

$$A_{\lambda} = 1.086 N_H \left( \frac{n_d}{n_H} \sigma_d(\lambda) \right).$$

(4)

The normalized selective extinction is

$$\xi(\lambda) = \frac{E(\lambda, V)}{E(B, V)},$$

(5)

where $E(B, V) = A_B - A_V$ is the one usually used.

$$\xi(V) = 0, \quad \xi(B) = 1, \quad \xi(\infty) = -\frac{A_V}{E(B, V)},$$

(6)

Note that the extinction vanishes at long wavelengths.
2. Measurements

Measurements (for thousands of stars in the solar neighborhood) yield:

\[ R = \frac{A_v}{B(B,V)} \approx 3.1 \quad \frac{dE(B,V)}{ds} \approx 0.6 \text{ mag kpc}^{-1}. \]  

(7)

Note: \( A_B \approx 1.3 A_V \) and 1 kpc corresponds to 2 mags of visual extinction.

These results correlate with measured hydrogen gas columns for lines of sight with moderate extinction using the Lyman \( \alpha \) line for \( H \) and the analog Lyman and Werner bands for \( H_2 \):

\[ N_H \equiv N(H) + 2N(H_2). \]  

(8)

\[ N_H \approx 5.9 \times 10^{21} \text{ cm}^{-2} \left( \frac{E(B,V)}{\text{mag}} \right), \]  

(9)

or

\[ N_H \approx 1.9 \times 10^{21} \text{ cm}^{-2} \left( \frac{A_{\lambda}}{\text{mag}} \right), \]  

(10)

From

\[ n_H = \frac{dN_H}{ds}, \]

these correlations imply

\[ n_H \approx 6 \times 10^{21} \frac{dE(B,V)}{ds} \text{ cm}^{-2} \text{ mag}^{-1}, \]

or

\[ n_H \approx 1.2 \text{ cm}^{-3}. \]  

(11)
Appendix 3
Poor Man’s Estimate of the Dust to Gas Ratio

3. Dust to Gas Ratio

Rearrange Eq. 4 using Eq. 10:

\[ A_V = 1.086 N_H \left( \frac{n_d}{n_H} \sigma_d(V) \right) = \left( 1.086 \right) \left( 1.9 \times 10^{21} \text{ cm}^{-2} \right) \left( \frac{A_V}{m_{\text{ag}}} \right), \]

or

\[ \left( \frac{n_d}{n_H} \sigma_d(V) \right) = 4.6 \times 10^{-22} \text{ cm}^{-2}. \]  \hspace{2cm} (12)

Guess \( a \) by invoking the usual requirement from scattering theory, \( k a \simeq 1 \), or \( \lambda \simeq a/(2\pi) \): \( a \sim 1000 \text{ Å} \) at \( V \). If we also take \( \sigma(V) \simeq 2\pi a^2 \), Eq. 12 yields

\[ \sigma(V) \simeq 6 \times 10^{-10} \text{ cm}^2. \]  \hspace{2cm} (13)

Substituting into equation 12 gives an estimate of the dust abundance:

\[ \left( \frac{n_d}{n_H} \right) = 8 \times 10^{-13}. \]  \hspace{2cm} (14)

If the dust particles are compact and spherical with internal density \( \rho_{\text{int}} \sim 3 \text{ gr cm}^{-3} \), the mass of a dust particle is

\[ m_d = \frac{4\pi}{3} \rho_{\text{int}} a^3 \sim 1.25 \times 10^{-14} \text{ gr}. \]  \hspace{2cm} (15)

We can estimate the dust to gas mass ratio from

\[ \frac{\rho_d}{\rho_g} = \frac{n_d m_d}{n_H m}. \]  \hspace{2cm} (16)

The mean gas mass per hydrogen nucleus \( m \) ranges from \( 1.3m_H \) for completely atomic to \( 2.3m_H \) for completely molecular regions. With \( a \simeq 0.1 \mu m, \rho_d = 3 \text{ gr cm}^{-3} \), and \( a = 1.3m_H \),

\[ \frac{\rho_d}{\rho_g} \sim 0.005. \]  \hspace{2cm} (17)