Lecture 13 Interstellar Magnetic Fields

1. Introduction
2. Synchrotron radiation
3. Faraday rotation
4. Zeeman effect
5. Polarization of starlight
6. Summary of results

References
Zweibel & Heiles, Nature 385 101 1997
Crutcher, Heiles & Troland, Springer LNP 614 155 2003
History of Interstellar Magnetic Fields

• Probably started in 1949 with the discovery of the polarization of starlight by interstellar dust.

• Quickly followed by the suggestion (Shklovskiy1953) that synchrotron radiation emission powered the Crab Nebula. The optical polarization was confirmed the following year by Russian astronomers.

• Searches for the Zeeman splitting of the 21-cm line were suggested by Bolton & Wild in 1957.

• After 50 years of study, measuring the interstellar magnetic field remains an important challenge to observational astronomy.
2. Synchrotron Radiation

References: Jackson Ch. 14
Rybicki & Lightman Ch. 6
Shu I Ch. 18

A key parameter is the \textit{cyclotron frequency} $\omega_B$. From relativistic mechanics of a particle with charge $q$ and rest mass $m_0$ in a uniform field $B$:

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad \beta = v / c$$

$$\frac{d\vec{v}}{dt} = \vec{\omega}_B \times \vec{v} \quad \vec{\omega}_B = \frac{1}{\gamma} \left( \frac{qB}{m_0c} \right)$$
The motion can be decomposed into a **drift motion** along the field and a **circular motion** perpendicular to the field:

\[ \vec{v} = \vec{v}_\parallel + \vec{v}_\perp. \]

In the absence of other forces, charged particles execute a net **helical motion** along magnetic field lines.

The circular motion is described by the rotating vector:

\[
\begin{align*}
\vec{r} &= a \left( \cos \omega_B t \, \hat{x} + \sin \omega_B t \, \hat{y} \right) = \Re \, a e^{-i \omega_B t} \left( \hat{x} + i \hat{y} \right) \\
\vec{v} &= \omega_B a \left( -\sin \omega_B t \, \hat{x} + \cos \omega_B t \, \hat{y} \right) = \Re \left( -i \omega_B \vec{r} \right) \\
\dot{\vec{v}} &= -\omega_B^2 \vec{r}.
\end{align*}
\]

One can also derive an expression for the gyroradius,

\[
a = \frac{c p_\perp}{|q| B} = \frac{p_\perp}{m_0 \omega_B} = \left( \frac{c}{\omega_B} \right) \left( \frac{v_\perp}{c} \right)
\]
The numerical value of the cyclotron frequency ($\gamma=1$) is

$$\omega_B = 17.59 \text{ MHz} \ B \text{ for electrons}$$

$$\omega_B = 9.590 \text{ kHz} \ B \text{ (Z/A) for ions}$$

We will find that a typical interstellar magnetic field is 5 $\mu$G, for which these numbers apply:

<table>
<thead>
<tr>
<th>particle</th>
<th>$2\pi/\omega_B$ (in s)</th>
<th>$c/\omega_B$ (in cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron</td>
<td>0.0714</td>
<td>$3.4 \times 10^8$</td>
</tr>
<tr>
<td>proton</td>
<td>131</td>
<td>$6.26 \times 10^{11}$</td>
</tr>
</tbody>
</table>

Small by interstellar standards, they validate the spiraling motion of electrons and ion along magnetic field lines,

**Question:** What about dust grains with $Z/A << 1$. 

Summary of Textbook Treatments of Synchrotron Radiation

Basis
- circular orbits, gyrofrequency $\omega_B$
- Lorentz factor $\gamma$
- helical pitch angle $\alpha$
- L-W retarded potentials
- relativistic limit $\gamma >> 1$

Results
1. Mean power

$$\frac{dP(\omega)}{d\omega} = \frac{2}{3} \frac{q^2}{c} (\omega_B \beta \sin \alpha)^2 \gamma^4$$

(relativistic Larmor formula)
Results (cont’d)

2. Angular distribution – searchlight in cone of half-angle $\alpha$ about $\mathbf{B}$ and width $1/\gamma$ (previous figure).

3. Frequency distribution -

\[
\frac{dP(\omega)}{d\omega} = CF\left(\frac{\omega}{\omega_c}\right)
\]

\[
\omega_c = \frac{2}{3} \gamma^3 \omega_B \sin \alpha
\]

\[
C = \frac{3}{2\pi} \left(\frac{q^2}{c^2}\right) \left(\frac{|q| B}{m_0 c}\right) \sin \alpha
\]

(F plotted vs. $\omega/\omega_c$ (Rybicki & Lightman Fig. 6.6))

4. Polarization $\sim 75\%$, integrated over the spectrum for fixed $\gamma$, leading to plane-polarized emission in the plane of the sky (perpendicular to the line of sight), the tell-tale signature of synchrotron emission.
5. Power law electron energy distribution – suggested by local cosmic rays (next lecture) and the *observed* frequency spectrum

\[
\frac{dN(\gamma)}{d\gamma} = D \gamma^{-p} \quad \Rightarrow \quad \frac{dP(\omega)}{d\omega} \propto \left(\frac{B \sin \alpha}{\omega}\right)^{\frac{p-1}{2}}
\]

The observations suggest \( p \sim 3 \)
Results of Synchrotron Observations

• The characteristics of synchrotron radiation, polarization and power-law spectra, are observed towards extragalactic radio sources as well as pulsars.

• They are particularly useful in tracing the direction of the magnetic fields in external galaxies, as illustrated for M51 and NGC 891 (figures below).

• The magnitude of the field cannot be determined without making assumptions about the electron density since the flux is proportional to $B^2n_e$. 
2.8 cm observations of polarized emission from the face-on galaxy M51. *The magnetic field directions follow the spiral arms.*

Synchrotron emission at 6 cm from edge-on galaxy NGC 891 showing *halo emission*.

Sukumer & Allen

*Magnetic fields as well as cosmic rays must extend above the disk of the galaxy.*
3. Faraday Rotation

In 1845 Faraday discovered that the polarization of an EM wave can change in a magnetic field. Following Lecture 9, we focus on the **index of refraction** of an astrophysical plasma using Shu Vol. I, Ch. 20. Combining Maxwell’s equations (for harmonic variations) with Newton’s equation for an electron with the Lorentz force yields the index

\[
    n_{\pm}^2 = 1 - \frac{\omega_{pl}^2}{\omega(\omega \mp \omega_B)}
\]

\[
    \omega_{pl} = \sqrt{\frac{4\pi n_e e^2}{m_e}} = 5.63 \times 10^4 \sqrt{n_e} \text{ Hz}, \quad \omega_B(e) = 17.6\left(\frac{B}{\mu G}\right) \text{ Hz}
\]

for circularly polarized radiation traveling in the direction of the field (z)

\[
    \vec{E} = \vec{E}_0 (\hat{x} \pm i\hat{y}) e^{i(k_{\pm} z - \omega t)} \quad k_{\pm} = n_{\pm} (\omega / c)
\]
After a plane polarized wave, \( \vec{E} = (\vec{E}_0 / 2)[(\hat{x} + i\hat{y}) + (\hat{x} - i\hat{y})] \), travels a distance \( z \), it can be represented as

\[
\vec{E} = (\vec{E}_0 / 2) \left[ (\hat{x} + i\hat{y})e^{i(k_x z - \omega t)} + (\hat{x} - i\hat{y})e^{i(k_x z - \omega t)} \right]
\]

with the right and left amplitudes propagating with different phases. After a little algebra, this becomes

\[
\vec{E} = \vec{E}_0 e^{i(Kz - \omega t)} (\hat{x} \cos \psi + \hat{y} \sin \psi)
\]

\[
\psi = \frac{1}{2} (k_+ - k_-)z = \frac{1}{2} (n_+ - n_-) \frac{\omega}{c} z \quad K = \frac{1}{2} (k_+ + k_-)
\]

where \( \psi \) is the angle through which the plane of polarization has been rotated. It involves the difference in the index of refraction:

\[
\Delta n = \frac{1}{2} \frac{\omega_{pl}^2}{\omega^2} \frac{\omega_B}{\omega} \quad \Rightarrow \quad \psi = \frac{\omega_{pl}^2}{\omega^2} \frac{eB}{2m_ec^2} z
\]
More generally for realistic & variable $B$ and $n_e$, the plane of polarization is rotated by

$$
\psi = 2\pi \frac{e^3}{m_e c^4} \lambda^2 \ RM, \quad RM \equiv \int ds \ n_e B_\parallel \approx L_\parallel < n_e > < B_\parallel >
$$

where $RM$ is the Faraday Rotation Measure. In practice, we cannot measure absolute phases (since we do not know the polarization produced by the source). Instead the wavelength dependence is used, on the assumption that the initial polarization is independent of wavelength.

$RMs$ are measured towards both extragalactic radio sources and pulsars (see the following figures). For pulsars, measurements of both $RM$ and $DM$ yield values of $B_\parallel$ for the WIM $\sim 5 \ \mu$G.
Figure 2. The distribution of the $RM$s of pulsars within $8^\circ$ of the Galactic plane. Positive $RM$s are shown as crosses, negative $RM$s as circles. The most recent $RM$ data are indicated by $X$ and open squares. The symbol sizes are proportional to the square root of $|RM|$, with the limits of 5 and 250 rad/m$^2$. The directions of the bisymmetric field model are given as arrows. The approximate location of four spiral arms is indicated as dotted lines. The dotted circle has a radius of 3 kpc (from Han et al., 1999a).

Pulsar $RM$s within $8^\circ$ of the galactic plane. Positive values $+$ negative values $\circ$
The field reversals are believed to occur in between the spiral arms. Below, the early results of Rand & Lyne (1994) are superimposed on the electron density model of Taylor & Cordes (1993) discussed in Lecture 9
Extragalactic RMs:
closed circles > 0
open circles < 0

$RMs$ can also be measured against extragalactic sources, but are more complicated to interpret because of the long lines of sight and contamination from the sources. The results indicate that the magnetic field has a substantial random component, especially above the plane.
4. Zeeman Measurements at 21 cm

Proposed by Bolton & Wild: 
ApJ 125 296 1957
in a short ApJ Note

Useful detailed discussion: 
Crutcher et al.

OH for molecular clouds, 
e.g., Crutcher & Troland 

Review: see Sec. 3 of 
Crutcher, Heiles, & Troland 
Springer LNP 614 155 2003
Physical Basis for Using the Zeeman Effect

Add the Zeeman splitting to the ground levels of HI:

<table>
<thead>
<tr>
<th>$E_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M=1</td>
</tr>
<tr>
<td>M=0 (F+1)</td>
</tr>
<tr>
<td>M=-1</td>
</tr>
<tr>
<td>M=0 (F=0)</td>
</tr>
</tbody>
</table>

For the magnetic fields of interest, the Zeeman splitting

$$E_z = \frac{e\hbar}{2m_e c} B = 1.400 \text{ MHz} \left( \frac{B}{G} \right)$$

is negligible compared to the broadening of the basic hfs splitting (1420 MHz).

Observing along the field, one sees only circular polarization (M=1 to 0 & M= -1 to 0). Perpendicular to the field, one sees plane polarization ($\pi$ parallel to the field and $\sigma$ perpendicular to the field).
Application of the 21-cm Zeeman Effect

If the 21-cm transition were not (thermally & turbulently) broadened, the polarization capability of radio receivers would resolve the Zeeman effect & measure the magnetic field.

If the field has a component along the line of sight, the broadened line wings will be circularly polarized, so the suggestion of Bolton & Wild was to compare the intensity difference between the line wings. This difference (Stokes parameter $V$) is related to the derivative of the unpolarized intensity $I$,

$$V = \frac{dI}{d\nu} \nu \cos \vartheta$$

where $\nu_Z$ is the Zeeman frequency and $\theta$ is the angle between $\mathbf{B}$ and the line of sight.
By measuring both $V$ and $dI/d\nu$, the parallel component of the magnetic field can be measured.

Example comparing the Stokes $V$ spectrum with the derivative of the unpolarized intensity.

Crutcher & Troland
OH in the core of L1544
Results of 21-cm Zeeman Measurements

Preliminary results for the diffuse CNM* (Heiles, Crutcher, & Troland 2003)

There is a lot of scatter. Non detections are not shown. The typical field is $B_\parallel = 5 \, \muG$.

*Results for dark cloud & cores will be discussed later.
Addendum on the Stokes Parameters

From Rybicki & Lightman, an EM wave can be expressed using either the observer’s x-y frame or the principle axes of a polarization ellipse. The angle $\beta$ gives the components in the former, and $\chi$ is the angle that the principle axis makes with the x-axis.

$$E_x = \mathcal{E}_0 (\cos \beta \cos \chi \cos \omega t + \sin \beta \sin \chi \sin \omega t)$$
$$E_y = \mathcal{E}_0 (\cos \beta \sin \chi \cos \omega t - \sin \beta \cos \chi \sin \omega t)$$

The Stokes parameters are then defined as:

$$I \equiv \mathcal{E}_1^2 + \mathcal{E}_2^2 = \mathcal{E}_0^2$$
$$Q \equiv \mathcal{E}_1^2 - \mathcal{E}_2^2 = \mathcal{E}_0^2 \cos 2\beta \cos 2\chi$$
$$U \equiv 2\mathcal{E}_1 \mathcal{E}_2 \cos (\phi_1 - \phi_2) = \mathcal{E}_0^2 \cos 2\beta \sin 2\chi$$
$$V \equiv 2\mathcal{E}_1 \mathcal{E}_2 \sin (\phi_1 - \phi_2) = \mathcal{E}_0^2 \sin 2\beta.$$  

Note that, in addition to the unpolarized intensity $I$, there are only two independent parameters (the direction of the polarization).

*All this is mathematical:* The problem is to express the results of a given polarization experiment in terms of these parameters. For the radio Zeeman measurements, see Crutcher et al. (1993)
5. Polarization of Starlight

$B_\perp$ for $\sim 10^4$ nearby stars, c.f. magnetic alignment of dust grains (discussed in a later lecture on magnetic fields and star formation).

1. The galactic field is “mainly in the plane”.
2. It is strong along the local spiral arm.
3. Random component is up tp 50% larger than uniform.
4. Out of plane features are associated with loops & regions of star formation.
6. Summary

Different techniques measure different things, e.g., field components parallel (RM & Zeeman) and perpendicular (synchrotron and dust polarization). Zeeman measurements are sensitive to neutral regions, and synchrotron emission is particularly useful for external galaxies. The results also depend on the sampling (averaging) volume.

For the Milky Way, the field is mainly parallel to the disk midplane, concentrated in the spiral arms with inter-arm reversals, and twice as large as the older value of 3 µG. The random and uniform components are the same order of magnitude. A rough value for the total field in the solar neighborhood is 6 µG. The field increases with decreasing galactic radius, and is about 10 µG at $R = 3$ kpc.
Summary (concluded)

*For external galaxies,* the synchrotron emission studies show the effects of spiral structure, the existence of thin and thick disk components, and occasionally inter-arm reversals. Typical field values are 10 µG, not all that different than the Milky Way.

For more on magnetic fields in external galaxies, see: R. Beck, SSR 99 243 2001 or astro-ph/0012402