7. Dust Grains & Interstellar Extinction

James R. Graham
University of California, Berkeley
Visual Extinction

• Presence of interstellar gas or nebulae has a long history
  – Existence of absorbing interstellar dust remained controversial

• Star counts show few stars in some directions (W. Herschel 1738-1822)
  – No stars or something hiding them?
  – No consensus until the 1930’s

• R. Trümpler (1930 PASP 42 214) conclusively demonstrated existence of interstellar absorption
  – Compare luminosity distances & angular diameter distances for open clusters
    • Angular diameter distances are systematically smaller
    • Discrepancy grows with distance
    • Distant clusters are redder
  – Estimated ~ 2 mag/kpc absorption
    • Attributed to Rayleigh scattering by very tiny grains ~ 25 Å radius
Evidence for Interstellar Dust

• Extinction, reddening and polarization of starlight
  – Dark clouds
• Reflection nebulae
  – Diffuse Galactic light
  – X-ray halos
• Continuum IR emission
  – Diffuse emission from the Galaxy
    • Correlated with HI and CO
  – Stars with IR excesses
• Depletion of refractory elements (e.g., Si, Fe, Ca) from interstellar gas
Barnard 68

Extincted stars appear red
Scattered Light in the Pleiades
A Dark Nebula

B 33
Nature of the Absorbers

• Before Trumpler it was known that Rayleigh scattering by gas cannot account for the magnitude of extinction—too much mass is required
  – Small solid particles absorb and scatter very efficiently

• The advent of photoelectric photometry (1940-1950) lead to the discovery and quantification of interstellar reddening
  – Precise comparison of colors of stars of the same spectral type (temperature)
  – $A_\lambda \sim \lambda^{-1.5}$ between 0.3 - 2.5 µm
    • Expected for dust with $2\pi a/\lambda \sim 1$, i.e., $a \sim 0.1$ µm

• Star light is polarized in regions of high extinction
  – One polarization state is selectively removed
    • Scattering by small conducting or dielectric particles
    • Not spheres, but elongated and aligned (by the Galactic B-field)
Interstellar Extinction

- Continuum opacity
  - Uniform shape
- General $\lambda^{-1.5}$ trend in the visible/near-IR
- Steep UV rise
  - Peak at $\sim 800$ Å
- Strong features
  - $\lambda = 220$ nm
    $\Delta \lambda = 47$ nm
  - $\lambda = 9.7$ μm
    $\Delta \lambda \sim 2-3$ μm
Reddening & Extinction

• Extinction law is deduced from observations of reddened stars of known spectral type ($M_\lambda$) and distance ($d$)

$$m_\lambda = M_\lambda + 5 \log d - 5 + A_\lambda$$

• The color excess, e.g.,

$$E(B-V) = (B-V) - (B-V)_0 = A(B)-A(V)$$
where the intrinsic color is $(B-V)_0$

• **Selective extinction**, $R_\lambda$, measures steepness of the extinction curve
  
  – $R_V = A(V) / [A(B)-A(V)] = A(V) / E(B-V)$
  
  • Steep in diffuse ISM: $R_V = 3.1\pm0.2$
  • Shallow in dark clouds: $R_V \approx 5$
  • If $R_\lambda$ is known (or assumed) then the observed color of a star of known spectral type can be converted to $A_\lambda$
The Reddening Vector

- Cluster at 100 pc
  - $A_V = 5$ mag.
  - $A_K = 0.54$ mag.
  - $A_V - A_K = 4.46$
“Standard” Interstellar Extinction
Gas-to-Dust Ratio

• For a constant *gas-to-dust ratio, Z*
  – *E(B-V) and A (V)* are correlated with distance and with H column, *N_H*
  – In the plane of the Milky Way the average reddening, *E(B-V)*
    = 0.61 mag. per kpc
  • *A_V = 1.9* mag. kpc⁻¹ for *R_V = 3.1*
• For uniform size grains, radius *a*

\[
A_V = 2.5 \log_{10}(e) \tau_V = 1.086 \pi a^2 Q_{ext} N_d
\]

\[
= 1.086 \frac{3 Q_{ext} Z m_H}{4 a \rho_d} N_H
\]

\[
\approx 0.3 Q_{ext} a^{-1}_5 \left(\frac{Z}{0.006}\right) \left(\frac{2.5 \text{g cm}^{-3}}{\rho_d}\right) N_{H,21} \text{mag.}
\]
Gas-to-Dust Ratio

- Compare $E(B-V)$ measurements with $N(\text{HI})$ & $N(\text{H}_2)$
  - $A \ (V) \approx N_H / 1.87 \times 10^{21} \text{ mag cm}^{-2}$ for $R_V = 3.1$

*Figure 2* Correlations between gas column densities and interstellar reddening for 100 stars from the *Copernicus* atomic and molecular hydrogen survey (Savage et al. 1977, Bohlin, Savage & Drake 1978): (a) shows the atomic hydrogen column density, $N(\text{HI})$, versus $E(B-V)$, (b) shows the total hydrogen column density, $N(\text{HI} + \text{H}_2) = N(\text{HI}) + 2N(\text{H}_2)$, versus $E(B-V)$. Be stars are denoted with the open symbols. The solid line in (a) gives the average atomic hydrogen to $E(B-V)$ ratio $4.8 \times 10^{21}$ atoms cm$^{-2}$ mag$^{-1}$. In (b) the solid line gives the average total hydrogen to $E(B-V)$ ratio of $5.8 \times 10^{21}$ atoms cm$^{-2}$ mag$^{-1}$. The point for $\rho$ Oph in (a) and (b) should be moved upward by about a factor of 2.7.
Scattering & Absorption Definitions

• Efficiency is given in terms of $Q_{\text{ext}}$, $Q_{\text{sca}}$, & $Q_{\text{abs}}$
  $Q_{\text{ext}} = Q_{\text{sca}} + Q_{\text{abs}}$

$$\sigma_{\text{abs}} = Q_{\text{abs}} \pi a^2$$

$$\sigma_{\text{sca}} = Q_{\text{sca}} \pi a^2$$

$$\sigma_{\text{ext}} = Q_{\text{ext}} \pi a^2 = (Q_{\text{abs}} + Q_{\text{sca}}) \pi a^2$$

albedo $= \frac{\sigma_{\text{sca}}}{\sigma_{\text{ext}}} = \frac{Q_{\text{sca}}}{Q_{\text{ext}}} \leq 1$

• In general $Q = Q(a, \lambda)$
Scattering & Absorption Definitions

• Optical depth and extinction efficiency

\[ \tau_{\lambda}^{ext} = \int n_{dust} \sigma_{\lambda}^{ext} ds = \sigma_{\lambda}^{ext} \int n_{dust} \]

\[ = \pi d^2 Q_{ext}(\lambda) N_{dust} \]

• Optical depth and extinction

\[ I(\lambda) = I_0(\lambda) \exp[-\tau_{\lambda}^{ext}] \]

\[ A_{\lambda} = -2.5 \log_{10}[I(\lambda)/I_0(\lambda)] \]

\[ = 2.5 \log_{10}(e) \tau_{\lambda}^{ext} = 1.086 \tau_{\lambda}^{ext} \]
Scattering & Absorption Definitions

- Scattering efficiency is a function of the angle, $\theta$, between the incident and scattered wave
  - Quantified by the **phase function**, $g$
  
  $$
g = \langle \cos \theta \rangle = \frac{\int_0^\pi I(\theta)\cos \theta d\Omega}{\int_0^\pi I(\theta) d\Omega}
  \]

  - Isotropic scattering $\langle \cos \theta \rangle = 0$
  - Forward scattering $\langle \cos \theta \rangle = 1$
  - Back scattering $\langle \cos \theta \rangle = -1$
Scattering & Radiation Pressure

- Light carries momentum as well as energy
  - Of the incident radiation $I_0$ the absorbed part $I_0 \pi a^2 Q_{\text{abs}}$ is entirely lost
  - A fraction, $g$, of the scattered energy is returned to the forward beam

- The total flux removed from the forward beam is $I_0 \pi a^2 (Q_{\text{abs}} + Q_{\text{sca}} - gQ_{\text{sca}})$
  - $Q_{\text{ext}} - gQ_{\text{sca}} \equiv Q_{\text{pr}}$ is the efficiency factor for radiation pressure
  - Forward momentum removed from the beam is $I_0 \pi a^2 Q_{\text{pr}}/c$
    and this is the radiation pressure on the grain
Mie Scattering and Q’s

- General theory of scattering by uniform spheres
  - Radius $a$
  - Refractive index $m = n - i \kappa$
    - In general $m = m(\lambda)$

- Solution to Maxwell’s equations found in closed form
  - Scattered wave is expressed in terms of spherical harmonics (angular part) and Bessel functions (radial)
    - Boundary conditions on $E$ and $B$ yield the solution for all space
  - For small $x = 2\pi a/\lambda$ retain only a few terms
    - Simple asymptotic forms for small grains/long wavelength
  - For large $x$ need to sum many terms
Asymptotic Mie Scattering Formula

- For small $x = 2\pi a/\lambda$

\[
Q_{abs} = -4x \text{Im} \left( \frac{m^2 - 1}{m^2 + 2} \right) \propto \lambda^{-1}
\]

\[
Q_{sca} = \frac{8}{3} x^4 \text{Re} \left\{ \left( \frac{m^2 - 1}{m^2 + 2} \right)^2 \right\} \propto \lambda^{-4}
\]

- cf. Rayleigh scattering

- Note: for small $x$

\[
\sigma_{abs} = Q_{abs} \pi a^2 \propto a^3 \propto m_{dust}
\]

absorption by grains depends only on total mass in grains
Scattering From a Raindrop

- $m = 1.33 - 0i$
  - $Q_{\text{ext}} \approx 4$ at $x \approx 6$
  - $x \to \infty$, $Q_{\text{ext}} = 2$
  - $Q_{\text{abs}} = 0$

- In general $m$ depends on wavelength
  - $x$ is a size parameter!
Angular Distribution \((m=1.33)\)
What’s This?

\[ m = 1.33 \]
\[ a = 100 \, \mu m \]
\[ \lambda = 600 \, \text{nm} \]
Here’s a Hint
Pure vs. Dirty Dielectrics

$m = 1.33 + 0.00 \, \text{i}$

$m = 1.33 - 0.05 \, \text{i}$

$Q_{\text{ext}} = Q_{\text{sca}}$

$x = 2 \, \pi a / \lambda$

$x = 2 \, \pi a / \lambda$
Nature of Interstellar Dust

- Gas-to-dust ratio
- Elemental composition
- Spectral features
- Grain size distribution
- Grain heating and cooling
- Carbon in the ISM
The Purcell Limit

• How much interstellar dust?

\[
\int_0^\infty Q_{\text{ext}} \, d\lambda = 4\pi^2 a \left( \frac{m^2 - 1}{m^2 + 2} \right)
\]

\[
\tau_{\text{ext}} = Q_{\text{ext}} \pi a^2 N_d, \quad N_d = \rho_d L / m_d
\]

\[
\int_0^\infty \tau_{\text{ext}} \, d\lambda = 4\pi^3 a^3 N_d \left( \frac{m^2 - 1}{m^2 + 2} \right)
\]

for *spherical grains*. Lower limit on grain volume for other shapes

• Extinction, \( A_\lambda \), is related to the dust column, \( N_d \)
Gas to Dust Ratio

- Convert from $\tau_\lambda$ to $A_\lambda$
  \[
  \int_0^\infty \frac{A_\lambda d\lambda}{L} = 3\pi^3 1.086 \left( \frac{m^2 - 1}{m^2 + 2} \right) n_d V_{gr}
  \]
  \[
  n_d V_{gr} = \frac{n_d m_{gr}}{m_{gr} / V_{gr}} = \frac{\rho_d}{\rho_{gr}} = \frac{\text{average dust density}}{\text{density of solid}}
  \]

- Standard extinction curve $A_\lambda$ (lower limit to integral)
- $A_V/L \rightarrow \text{mean dust density}$
- $N_H/A_V \rightarrow \text{gas-to-dust ratio}$
- Silicate dust
  - $m = 1.5 - 0i$
  - $\rho_{gr} = 2.5 \text{ g cm}^{-3}$

\[
\text{gas-to-dust ratio} = \frac{\rho_d}{\rho_{gas}} \approx 0.006
\]
Implications of the Purcell Limit

• Significant fraction of heavy elements in interstellar grains
  – Unless grains are very non-spherical and conducting
  – Mass fraction of heavy elements, $Z = 0.017$
  – ~ 40% of “metals” are in grains

• Typical model:
  – Silicate grains
    – Mg, Si, Fe and O (20-95%) in $(\text{Mg,Fe})_2\text{SiO}_4$
  – Carbonaceous material (graphite & organics)
    • C (60%)
  – Some SiC
### Interstellar Dust Composition

<table>
<thead>
<tr>
<th>Element</th>
<th>Abundance</th>
<th>A</th>
<th>( M/M_H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>( 0.6 \times 2.5 \times 10^{-4} )</td>
<td>12</td>
<td>0.0018</td>
</tr>
<tr>
<td>Mg</td>
<td>( 3.4 \times 10^{-5} )</td>
<td>24</td>
<td>0.0008</td>
</tr>
<tr>
<td>Fe</td>
<td>( 2.8 \times 10^{-5} )</td>
<td>56</td>
<td>0.0016</td>
</tr>
<tr>
<td>Si</td>
<td>( 3.2 \times 10^{-5} )</td>
<td>28</td>
<td>0.0009</td>
</tr>
<tr>
<td>O</td>
<td>( 0.2 \times 4.6 \times 10^{-4} )</td>
<td>16</td>
<td>0.0014</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>0.0066</td>
</tr>
</tbody>
</table>
Clues in the Extinction Curve

\[ A_\lambda \text{ [mag]} \]

\[ \lambda [\mu m] \]

220 nm

9.6 \(\mu m\)
Interstellar Extinction Curve

• The shape of the interstellar extinction curve reveals the makeup of dust
  – Overall smoothness of $A_{\lambda}$ implies multi-component

• Size distribution of grain
  – General variation of extinction with wavelength

• Composition
  – Discrete features in the extinction curve
    • 220 nm bump
    • 9.7 & 18 µm
Vibrational Modes of Silicate Minerals

<table>
<thead>
<tr>
<th>Species</th>
<th>Mode</th>
<th>Wavelength (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MgSiO$_3$</td>
<td>Si-O stretch</td>
<td>9.7</td>
</tr>
<tr>
<td></td>
<td>O-Si-O bend</td>
<td>19.0</td>
</tr>
<tr>
<td>Mg$_2$SiO$_4$</td>
<td>Si-O stretch</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td>O-Si-O bend</td>
<td>19.5</td>
</tr>
<tr>
<td>FeSiO$_3$</td>
<td>Si-O stretch</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td>O-Si-O bend</td>
<td>20.0</td>
</tr>
<tr>
<td>Fe$_2$SiO$_4$</td>
<td>Si-O stretch</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>O-Si-O bend</td>
<td>20.0</td>
</tr>
<tr>
<td>SiC</td>
<td>SiC stretch</td>
<td>11.2</td>
</tr>
</tbody>
</table>
Silicate Minerals

- Silicate minerals generally have strong absorption resonances near 10 µm due to the Si-O bond stretch
  - Virtually certain that the interstellar 9.7 µm feature is due to absorption by interstellar silicate material
  - 10 µm emission feature is observed in outflows from cool O-rich stars
    - Expected to condense silicate dust
    - Absent in the outflows from C-rich stars where silicates do not form because all of the O is locked up in CO

- Broad feature at 18 µm is presumed to be the O-Si-O bending mode in silicates
The 220 nm Feature

- The 220 nm feature is ubiquitous in the Milky Way
  - $217.5 \pm 0.5$ nm
  - Variation in the width (10%) and strength
- Graphite has a strong UV resonance due to $\pi$-orbital valence electrons
  - Why is the feature so uniform?
- 220 nm bump is weak in the Small Magellanic Cloud
  - Weakness correlated with C/O
- $\text{Mg}_2\text{SiO}_4$ grains contaminated by OH$^-$