Part 3 Blackboard Review with Slides

1. Spectroscopy Of Cool Regions
2. Interstellar Chemistry
3. Molecular Clouds
Potential Energy Curves for H₂

Lyman band absorption followed by dissociation into the ground state continuum

donation
Energy Levels of CO
Summary of CO Diagnostic Lines

**UV** – CO is something like a heavy H₂: The UV bands occur in the far UV near 1000 Å with oscillator strengths of the same order of magnitude (10⁻²) as for H₂. They have have been detected in absorption towards diffuse clouds where the CO maximum abundance is 10⁻⁵.

**NIR** - \( \Delta v = 1 \) transitions define the *fundamental* bands

\( \Delta v = 2 \) transitions define the *1st overtone bands*

- \( 4 \rightarrow 1 \) \( \Delta v = 1 \) transition: 6859 cm⁻¹
- \( 3 \rightarrow 1 \) \( \Delta v = 1 \) transition: 4260
- \( 2 \rightarrow 1 \) \( \Delta v = 1 \) transition: 2143
- \( 1 \rightarrow 0 \) \( \Delta v = 1 \) transition: 0

\( \Delta v = 2 \) transitions:

- \( 4 \rightarrow 2 \): 4.6 μm
- \( 3 \rightarrow 2 \): 2.3 μm

\( A(1-0) \sim 37 \text{ s}^{-1} \)
The Ground Level CO rotational bands

\[ E_J = J(J+1)B, \quad \Delta E_J = E_J - E_{J-1} = 2BJ \quad \text{for small } J \]

\[ B = 1.922529 \text{ cm}^{-1}, \quad B/k_B = 2.766 \text{ K} \]

\[ A_{J,J-1} = \frac{3J^4}{2J+1} A_{10}, \quad A_{10} = 7.17 \times 10^{-8} \text{ s}^{-1} \]

\[ f_{J-1,J} = \frac{J^2}{2J-1} f_{01}, \quad f_{01} = 2.8 \times 10^{-9} \]

approximate collisional rate coefficient:

\[ k_{J',J}(\text{H}_2+\text{CO}) \sim 10^{-11} T^{0.5} \text{ cm}^3\text{s}^{-1} \]

for \(|J-J'| = 1.2|
CO rotational bands (cont’d)

Frequencies in GHz

<table>
<thead>
<tr>
<th></th>
<th>CO</th>
<th>$^{13}$CO</th>
<th>$^{18}$O</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-0</td>
<td>115.271</td>
<td>110.201</td>
<td>219.6</td>
</tr>
<tr>
<td>2-1</td>
<td>230.518</td>
<td>220.4</td>
<td>219.6</td>
</tr>
<tr>
<td>3-2</td>
<td>345.796</td>
<td>330.6</td>
<td>329.4</td>
</tr>
</tbody>
</table>

See NIST /MolSpec/Diatomic

Critical Densities

$$n_{cr}(1-0) \approx \frac{7167}{T^{0.5}} \text{ cm}^{-3} = 1.43 \times 10^3 \left(\frac{25}{T}\right)^{1/2} \text{ cm}^{-3}$$

<table>
<thead>
<tr>
<th>$J \rightarrow (J-1)$</th>
<th>$n_{cr}(J \rightarrow (J-1))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-0</td>
<td>$1.4 \times 10^3$</td>
</tr>
<tr>
<td>2-1</td>
<td>$2.3 \times 10^3$</td>
</tr>
<tr>
<td>3-2</td>
<td>$7 \times 10^4$</td>
</tr>
</tbody>
</table>

The fundamental transition at 2.6 mm is easily thermalized, especially when line trapping is considered.
Optical Depth of the CO Rotational Lines

Start with the standard formula

\[
\tau_{J,J-1} = f_{J,J-1} \frac{\pi e^2}{m_e c} \frac{\lambda_{J,J-1}}{b} N_{J-1}(\text{CO}) \left( 1 - \frac{P_J / 2J + 1}{P_{J-1} / 2J - 1} \right)
\]

including the stimulated emission factor, needed because the transition and thermal are about the same (10 K):

\[
1 - \frac{P_J / 2J + 1}{P_{J-1} / 2J - 1} \approx 1 - \exp(-J 2.766 K / T)
\]

For a constant CO abundance, the columns are

\[
N_J = x(\text{CO}) P_J N_H
\]

For a thermal population,

\[
P_J \approx \frac{1}{Z} (2J + 1) e^{-J(J+1)B/kT}, \quad Z \approx \frac{T}{B}
\]
CO rotational line optical depth

Putting all the pieces together leads to:

\[
\tau_{J-1,J} \approx f_{J-1,J} \frac{\pi e^2}{m_e c} \frac{\lambda_{J,J-1}}{b} \chi(\text{CO}) n_H \left[ \frac{2J - 1}{T / B} e^{-J(J-1)B/T} (1 - e^{-2BJ/T}) \right]
\]

\[
= 930 \, J \, x(\text{CO})_4 \frac{10K}{T} \frac{\text{km s}^{-1}}{b} \frac{N_H}{A_v} \left[ e^{-J(J-1)B/T} (1 - e^{-2BJ/T}) \right]
\]

**CO(1-0) and its other low-J transitions are very optically thick.**

Consider the critical density of the CO(1-0) transition:

\[
n_{cr} \approx \frac{\beta_{10} A_{10}}{k_{10}} \approx \frac{A_{10}}{k_{10}} \frac{1}{\tau_{10}} = 2.3 \times 10^3 \text{cm}^{-3} \left( \frac{10K}{T} \right)^{1/2}
\]

Large \( \tau_{10} \) reduces the critical density.
Formation and Destruction of CO

Formation in Cool Regions: Fast ion-molecule reactions
\[ k \approx 10^{-9} \text{ cm}^3 \text{ s}^{-1} \]

\[
\begin{align*}
O^+ + H_2 & \rightarrow OH^+ + H \\
OH^+ + H_2 & \rightarrow OH_2^+ + H \\
OH_2^+ + H_2 & \rightarrow OH_3^+ + H \\
H_3^+ + O & \rightarrow OH^+ + H_2 \\
H_3^+ + OH & \rightarrow OH_2^+ + H_2 \\
H_3^+ + H_2O & \rightarrow OH_3^+ + H_2
\end{align*}
\]

and fast dissociative recombination, \[ \beta \approx 10^{-9} \text{ cm}^3 \text{ s}^{-1} \]:

\[
\begin{align*}
OH^+ + e & \rightarrow O + H \\
OH2^+ + e & \rightarrow OH + H \quad \text{and} \quad O + H_2 \text{ or } 2H \quad 22\% \text{ OH} \\
OH3^+ + e & \rightarrow H_2O + H \quad \text{and} \quad OH + H_2 \text{ or } 2H, \text{ etc.} \quad 75\% \text{ OH}
\end{align*}
\]

OH is a crucial intermediary in the formation of CO, and it requires \( H_2 \) as a precursor. But the synthesis of OH is driven by the ions \( O^+ \) and \( H_3^+ \). This scheme also produces \( H_2O \) and other O-bearing molecules.
Early CO map of Orion from the Goddard-Columbia 1.2 m telescope plus Blaauw’s dated sub-associations

1a - 12 Myr
1b - < 1 Myr
Trapezium - < 1 Myr
1c - 2 Myr

The Trapezium cluster has about 3500 young stars, including a handful of O,B stars
Taurus-Perseus-Auriga Complex
### Approximate Typical Properties of Local GMCs

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>4000</td>
</tr>
<tr>
<td>Mass</td>
<td>$2 \times 10^5 , M_\odot$</td>
</tr>
<tr>
<td>Mean diameter</td>
<td>45 pc</td>
</tr>
<tr>
<td>Projected surface area</td>
<td>2000 pc$^2$</td>
</tr>
<tr>
<td>Volume</td>
<td>$10^5 , \text{pc}^3$</td>
</tr>
<tr>
<td>Volume density ($\text{H}_2$)</td>
<td>300 cm$^{-3}$</td>
</tr>
<tr>
<td>Mean surface density</td>
<td>$1.5 \times 10^{22} , \text{cm}^{-2}$</td>
</tr>
<tr>
<td>Surface density</td>
<td>4 kpc$^{-2}$</td>
</tr>
<tr>
<td>Mean separation</td>
<td>500 pc</td>
</tr>
</tbody>
</table>
X-factor Method 5: HI/IRAS/CO

- Dame et al. (ApJ 547 792 2001) used IRAS far-IR emission as a tracer of total gas column density
  - Calibrated with the Leiden-Dwingeloo 21-cm HI survey in regions free of CO emission
  - Total gas map was differenced with the a HI map to obtain a complete and unbiased predicted map of H$_2$
    - Close agreement between this map and observed CO implies that few molecular clouds at $|b| < 30$ have been missed by the present CO survey
  - The ratio of the observed CO map to the predicted molecular map provides a measure of the local average X-factor for $|b| > 5^\circ$:

$$N(H_2)/I_{\text{CO}} = 1.8 \pm 0.3 \times 10^{20} \text{ cm}^{-2} \text{ K}^{-1} /\text{km s}^{-1}$$
## CO/H$_2$ Conversion Factor: Summary

<table>
<thead>
<tr>
<th>Source</th>
<th>$X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early work</td>
<td>2-5</td>
</tr>
<tr>
<td>$\gamma$-rays Hunter et al. 1997</td>
<td>1.56 ±0.05</td>
</tr>
<tr>
<td>HI/IRAS/CO Dame et al. 2001</td>
<td>1.8±0.3</td>
</tr>
<tr>
<td>$A_V$ Lada et al. 2003</td>
<td>~ 4</td>
</tr>
</tbody>
</table>

Units for $X$: $10^{20}$ cm$^{-2}$ / K km s$^{-1}$
Structure of Molecular Clouds

CO maps show that molecular gas is heterogeneous.

What is the topology of molecular clouds?
Is it useful to talk about discrete structure?

Blitz & Williams discuss three levels of structure: **clouds, clumps, and cores**, illustrated by the following maps of the Rosette Molecular Cloud in CO, C^{18}O, & CS:

*Figure 4*. Hierarchical cloud structure. The three panels show a representative view from cloud to clump to core. The bulk of the molecular gas (cloud; left panel) is best seen in CO which, although optically thick, faithfully outlines the location of the H\textsubscript{2}. Internal structure (clumps; middle panel) is observed at higher resolution in an optically thin line such as C^{18}O. With a higher density tracer such as CS, cores (right panel) stand out. The observations here are of the Rosette molecular cloud and are respectively, Bell Labs (90''), FCRAO data (50''), and BIMA data (10'').
Probes of Higher Density

Although the CO isotopes are useful for measuring the properties of widely distributed molecular gas, other probes are needed for localized high-density regions, especially cloud cores that give birth to stars. The solution is to use high-dipole moment molecules which have large critical densities.

\[
A_{ul} = \frac{64\pi^4}{3h} \mu_{ul}^2 \nu_{ul}^3
\]

<table>
<thead>
<tr>
<th>Species</th>
<th>(\mu) (D)</th>
<th>(\nu_{10}) (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO</td>
<td>0.110</td>
<td>115.3</td>
</tr>
<tr>
<td>CS</td>
<td>1.96</td>
<td>48.99</td>
</tr>
<tr>
<td>HCN</td>
<td>2.98</td>
<td>89.09</td>
</tr>
<tr>
<td>HCO⁺</td>
<td>3.93</td>
<td>89.19</td>
</tr>
<tr>
<td>(\text{N}_2\text{H}⁺)</td>
<td>3.4</td>
<td>93.18</td>
</tr>
</tbody>
</table>

Summary of Core Properties

1. associated with star formation, e.g., 50% or more have embedded protostars (SPITZER finding more)

2. elongated (aspect ratio ~ 2:1)

3. internal dynamics may be dominated by thermal or turbulent motion,

\[ \Delta v_{\text{FWHM}}^2 = \Delta_{\text{turb}}^2 + 8 \ln 2 \left( \frac{kT}{m} \right) \]

e.g., equally split for NH$_3$ cores. Note that

\[ \Delta v_{\text{th}} = \sqrt{8 \ln 2 \left( \frac{kT}{m} \right)} = 0.675 \text{ km s}^{-1} \sqrt{\frac{m_{\text{H}}}{m} \left( \frac{10K}{T} \right)} \]

is typically ~ 0.1 – 0.2 km s$^{-1}$
First Summary of Core Properties

4. temperature: $T \sim 10 - 20$ K

5. size: $R \sim 0.1$ pc

6. ionization: $x_e \sim 10^{-7}$

7. size-linewidth relation
   \[ R \sim \sigma^p, \quad p = 0.3-0.7 \]

8. approximate virial equilibrium

9. mass spectrum: similar to GMCs.

Figure 6. The mass spectrum of dense cores in the L1630 cloud from the study of LHS. The spectrum can be characterized by a power law with an index of -1.6.

fact that different definitions of clumps were employed in each case [20, 122].
Non-thermal vs. Thermal Line Widths

Figure 6. Nonthermal and thermal line widths in NH$_3$ dense cores. The FWHM of the distribution of nonthermal motions, $\Delta v_{NT}$, increases rapidly with the corresponding FWHM of the distribution of thermal motions of the molecule of mean mass, $\Delta v_T$. Thus turbulent cores tend to be warm, and quiescent cores tend to be cool (JMA).