0. Reading: Ch. 11 of Ryden

1. No More Jeans Swindle: an Expanding Medium

By combining the unperturbed Euler and Poisson equations for an expanding non-relativistic fluid, show that one no longer has to invoke Jeans swindle, and instead, you obtain

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\rho,$$

where $a$ is the scale factor of an isotropic and homogeneous expanding universe and $\rho$ is the average (i.e. unperturbed) density of the universe. Compare this equation with the equation for $\ddot{a}$ that we derived in class near the beginning of the semester. If there is any difference, comment on the origin of the difference.

2. Mass versus Light

One of the primary goals of every galaxy survey is to measure the distribution of the luminosities of galaxies, which has been found to be well parameterized by the luminosity function

$$\phi(L) dL = \phi_s \left( \frac{L}{L_*} \right)^\alpha e^{-L/L_*} d\left( \frac{L}{L_*} \right),$$

where $\phi(L)dL$ is the number density of galaxies with luminosity between $L$ and $L + dL$, and $\phi_s$, $L_*$, and $\alpha$ are parameters to be determined from galaxy surveys.

(a) Show that the luminosity density of galaxies, $j$, is related to $\phi_s$, $L_*$, and $\alpha$ by a simple algebraic expression. Derive this expression. (Hint: If your answer contains an integral, you haven’t tried hard enough.)

For all your answers below, don’t assume a specific value for $h$; instead, make sure to keep track of factors of $h$.

(b) A recent galaxy survey measures $\phi_s = 0.0165 h^3 \text{Mpc}^{-3}$, $\alpha = -1.13$, and $L_* = 1.26 \times 10^{10} h^{-2} L_\odot$. Calculate $j$. What fraction of this luminosity density is contributed by galaxies fainter than $L_*$?

(c) The classical technique for quantifying the amount of dark vs. luminous matter is to determine the mass-to-light ratio, $M/L$ (in units of solar $M_\odot/L_\odot$). Assuming the luminosity density from part (b) is representative for the universe (current galaxy surveys cover a large volume, so this should be an OK assumption), calculate the value of $M/L$ (in units of $M_\odot/L_\odot$) required to close the universe (i.e. $\Omega_m = 1$).
The value of $M/L$ is typically measured to be a few 10s in individual galaxies and a few 100s in galaxy clusters. Is there enough dark matter to make $\Omega_m = 1$?

3. The Milky Way’s Dark Matter Halo

The Milky Way’s rotation curve is approximately a constant, $v(r) \sim 220 \text{ km/s}$, for $5 < r < 20 \text{ kpc}$, This constant circular velocity cannot be explained by the observed distribution of luminous mass. Instead, we propose that the Milky Way is embedded in a roughly spherical dark matter halo.

(a) Calculate $M(r)$, the enclosed mass of the spherical dark matter halo as a function of $r$ in the range $5 < r < 20 \text{ kpc}$, assuming the contribution from luminous matter is negligible.

(b) Let us model the density profile of the dark matter halo with a simple power-law polynomial

$$\rho(r) = \rho_0 \left( \frac{r}{r_0} \right)^n$$

where $r_0 = 8 \text{ kpc}$ is the distance of the Sun from the center of the Milky Way. Compute $\rho_0$ and $n$.

(c) What is the density of the dark matter halo in the neighborhood of the Sun? What is the ratio of this density to the critical density, $\rho_c$ (assume $h = 0.7$)? Do you expect such a high value?

(d) Could $\rho(r)$ maintain the same power-law behavior out to infinity? Why or why not? What does this imply about the behavior of $v(r)$ at large radius?

(e) Let us assume that the dark matter halo is made up of grains of mass $m_g$ moving at a speed $v \sim 220 \text{ km/s}$. Estimate how many dark matter grains would hit the Earth every second (leave your answer as a function of $m_g$).