Star Track 2 - The Search for a Supermassive Black Hole

Early radio astronomers detected an immensely powerful source of radio waves towards the center of the Galaxy in the constellation Sagittarius; this mysterious object was designated SgrA*. More recently, infrared astronomers using adaptive optics have imaged individual stars near this object and tracked their motion with time. The observed orbit for one such star is plotted at right (also see CS-274 in the course reader; this star is "SO-2").

1. Make a rough estimate of the semimajor axis of the star’s orbit in AU using the diagram. The semimajor axis is defined as the distance from the center of the ellipse to the edge along the ellipse’s long axis. (This is an underestimate because the orbit is inclined about 45 degrees to the line of sight, but we’ll ignore the inclination in this exercise.)

2. Now estimate the period by comparing the year of the star’s closest approach to SgrA* to the year it was furthest away (half an orbit). The year it was furthest away is not quite the top of the ellipse because of the inclination of the orbit.

3. Before we do any calculations, stop and think a bit. Pluto has a semimajor axis of about 40 AU and a period of 250 years. But SO-2 has a huge semimajor axis and yet is whipping around the central object in only a few years. What does this tell us?

4. Kepler’s 3rd law says that:

   \[ P^2 = \frac{4\pi^2}{G(m_1 + m_2)} R^3 \]

   Here \( P \) is the period, \( R \) is the semimajor axis, and \( m_1 \) and \( m_2 \) are the masses of the objects.

   Use ratios to get rid of the constants, using the properties of the Earth’s orbit around the Sun (\( P_E, R_E, \) and \( m_{\text{Sun}} \)). Then put the equation in terms of \( m_1/m_{\text{Sun}} \) (assume \( m_2 \ll m_1 \)).

   \[ \frac{m_1}{m_{\text{Sun}}} = \]

5. Now calculate the mass of SgrA* in Solar masses using the formula above. Remember that \( P_E = 1 \) yr and \( R_E = 1 \) AU. (This won't exactly agree with the number given in class, but we're just doing a rough calculation.)

6. The closest approach of SO-2 to SgrA* is only 120 AU, so SgrA* must be smaller than that. Calculate the minimum density of SgrA* (in solar masses per cubic AU) for it to be concentrated in this small region.

7. Why do astronomers think this makes SgrA* a black hole, and not some other kind of object?