Radioactive Decay

In class we have talked about the decay of unstable elements in terms of the equation $e^{-\lambda t}$, which is the form that comes most naturally out of the mathematical equations for radioactive decay. However, there are other ways to express the equation, in particular in terms of the "half-life".

1. The half-life, denoted by $t_{1/2}$, indicates the amount of time for half of a sample to decay from the parent isotope to its daughter isotope. Suppose we have a sample of 1 million atoms with a half-life of one year. How many will remain after two years? After three years? After four years? After fifteen years?

2. What about after one and a half years? Write down an equation to determine how many atoms will be left of this element at any arbitrary time t.

3. Generalize the equation you wrote down in (2) for any sample of any element. Represent the initial number of atoms as N_i and the element's half-life as $t_{1/2}$.

4. You may remember, from that Algebra II or Pre-Calculus class you took but long forgot about, that you can change the base of an exponential without too much trouble. With your GSI's help (if necessary), convert the formula in (3) to the base-*e* version we used in class. What is λ expressed in terms of $t_{1/2}$?

Facts that might be useful:

$(x\wedge y)^z = x^{yz}$
$e^{\ln x} = x$
$(1/x)^y = x^{-y}$

5. Use the formula from class, and a bit more algebra, to write down an equation for the age a sample of some radioactive element.

Useful fact: $\ln(e^x) = x$

6. What information do we need to actually do the calculation in part (5) in real life? How do we determine that information?

Original worksheet by D. Perley

7. Below is a list of *every* radioactive element with a half-life of greater than 1,000 years, and its qualitative abundance in nature relative to other elements of the same atomic mass – data that can easily be looked up online by anyone with a computer. What does this tell you about the age of the Earth?

8. Can you explain the presence of a small number of elements with short half-lives that still seem to be present in nature?

9. Find some elements in the table below that are actually used to date materials. Why these elements, and not others?

isotope	half-life(yr)	abundance	isotope	half-life(yr)	abundance
Holmium-166m	1,200	none	Palladium-107	6,500,000	none
Berkelium-247	1,380	none	Hafnium-182	9,000,000	none
Radium-226	1,600	trace	Lead-205	15,300,000	none
Molybdenum-93	4,000	none	Curium-247	15,600,000	none
Holmium-153	4,570	none	Uranium-236	23,420,000	none
Curium-246	4,730	none	Niobium-92	34,700,000	none
Carbon-14	5,730	trace	Plutonium-244	80,800,000	none
Plutonium-240	6,563	none	Samarium-146	103,000,000	none
Thorium-229	7,340	none	Uranium-236	234,200,000	none
Americium-243	7,370	none	Uranium-235	703,800,000	rare
Curium-245	8,500	none	Potassium-40	1,280,000,000	rare
Curium-250	9,000	none	Uranium-238	4,468,000,000	common
Tin-126	10,000	none	Rubidium-87	4,750,000,000	common
Iodine-129	15,700	none	Thorium-232	14,100,000,000	common
Niobium-94	20,300	none	Lutetium-176	37,800,000,000	rare
Plutonium-239	24,110	none	Rhenium-187	43,500,000,000	common
Proactinium-231	32,760	trace	Lanthanium-138	105,000,000,000	rare
Lead-202	52,500	none	Samarium-147	106,000,000,000	common
Lanthanium-137	60,000	none	Platinum-190	650,000,000,000	rare
Thorium-230	75,380	none	Tellurium-123	>1 x 10^13	rare
Nickel-59	76,000	none	Osmium-184	>5.6 x 10^13	rare
Thorium-230	77,000	trace	Gadolinium-152	1.08 x 10^14	rare
Calcium-41	103,000	none	Tantalum-180m	>1.2 x 10^15	rare
Neptunium-236	154,000	none	Xenon-124	>1.6 x 10^14	rare
Uranium-233	159,200	none	Indium-115	4.41 x 10^14	common
Rhenium-186m	200,000	none	Zinc-70	>5 x 10^14	rare
Technetium-99	211,000	none	Hafnium-174	2.0 x 10^15	rare
Krypton-81	229,000	none	Osmium-186	2.0 x 10^15	common
Uranium-234	245,500	trace	Samarium-149	>2 x 10^15	common
Chlorine-36	301,000	none	Neodymium-144	2.29 x 10^15	common
Curium-248	340,000	none	Samarium-148	7 x 10^15	common
Bismuth-208	368,000	none	Cadmium-113	7.7 x 10^15	common
Plutonium-242	373,300	none	Cerium-142	>5 x 10^16	common
Aluminum-26	717,000	none	Tungsten-183	>1.1 x 10^17	common
Selenium-79	1,130,000	none	Vanadium-50	1.4 x 10^17	rare
Iron-60	1,500,000	none	Lead-204	1.4 x 10^17	common
Beryllium-10	1,510,000	none	Chromium-50	>1.8 x 10^17	common
Zircon-93	1,530,000	none	Tungsten-184	>3 x 10^17	common
Curium-247	1,560,000	none	Calcium-48	>6.3 x 10^18	common
Gadolinium-150	1,790,000	none	Molybdenum-100	1.0 x 10^19	common
Neptunium-237	2,144,000	none	Neodynium-150	>1.1 x 10^19	common
Cesium-135	2,300,000	none	Zircon-96	>3.8 x 10^19	common
Technetium-96	2,600,000	none	Selenium-82	1.1 x 10^20	common
Dysprosium-154	3,000,000	none	Tellurium-130	7.9 x 10^20	common
Bismuth-310m	3,040,000	none	Xenon-136	>2.4 x 10^21	common
Mietnerium-53	3,740,000	none	Tellurium-128	2.2 x 10^24	common
Technetium-98	4,200,000	none	Stable Isotopes	infinite	**

** - Of 280 total stable isotopes, about 70% are common, 30% are rare, and 1% are trace. There are no stable isotopes that are absent in nature.