Problem Set #5

Due Friday, November 3 at 5pm

Please indicate the time spent on the assignment and list any collaborators. You can submit the assignment by emailing a single PDF to dressing@berkeley.edu or by placing a hard copy in my mailbox or under my door (Campbell 605E).

1. Clouds (adapted from* Planetary Sciences *by de Pater & Lissauer)

The base of the methane cloud in Uranus's atmosphere is at a pressure level of 1.25 bar and temperature of 80 K. The saturation vapor pressure curve is given by the Clausius-Capeyron equation of state:

$$P_{\rm V} = C_{\rm L} e^{-L_{\rm S}/(R_{\rm gas} T)} \tag{1}$$

where $C_{\rm L}$ is a constant, $L_{\rm S}$ is the latent heat, and $R_{\rm gas} = 8.3145 \times 10^7$ erg deg^{-1} mole^{-1} is the gas constant. For this problem, $C_{\rm L} = 4.658 \times 10^4$ bar and $L_{\rm S} = 9.71 \times 10^{10}$ erg mole^{-1}.

a. Remembering that the mixing ratio can be related to the gas partial pressure, derive the CH_4 volume mixing ratio in Uranus's atmosphere. Express your answer as the ratio of the number density of CH_4 molecules to the total number density of the atmosphere.

b. Assuming that the atmosphere of Uranus is 83% H_2 and 15% He, determine the ratio of the number density of CH_4 to the number density of H_2.

c. When quoting abundances, astronomers often use funky units where the abundance of H is defined as $\log \epsilon_{\rm H} = 12.00$ and the abundances of other elements are given as $\log \epsilon_{\rm X} = \log(N_{\rm x}/N_{\rm H}) + 12$, where $N_{\rm X}$ and $N_{\rm H}$ are the number densities of elements X and H, respectively). Using these units, Asplund et al. (2009, ARAA, 47, 481) reported that the abundance of C in the solar photosphere is $\log \epsilon_{\rm C} = 8.43$. Convert the Asplund number into a C/H ratio and comment on how the CH_4 volume mixing ratio in Uranus's atmosphere compares to the solar C/H ratio.

2. Atmospheric Escape (adapted from* Exoplanet Atmospheres *by Sara Seager)

a. Compute the escape velocity $v_{\rm esc}$ of the Earth, Venus, Mars.

b. Compute the escape energies $E_{\rm esc}$ of H and O and the Jeans escape parameter λ_c. Recall that

$$E_{\rm esc} = \frac{1}{2} m v_{\rm esc}^2 \tag{2}$$

and

$$\lambda_c = \frac{G M_p m}{k T r_c} \tag{3}$$

where G is the gravitational constant, M_p is the mass of the planet, m is the particle mass, k is Boltzmann's constant, T is the exobase temperature, and r_c is the radial distance from the planet's center. Assume that the exobase temperatures are 900K for Earth, 270K for Venus, and 220K for Mars.

c. Assuming that the number density at the Earth's exobase is $N_{\rm ex} = 10^5$ cm^{-3}, estimate the rate of H escape by thermal evaporation (in atoms cm^{-2} s^{-1}) by using the Jeans formula

$$\Phi_J = \frac{N_{\rm exo} v_0}{2\sqrt{\pi}} \left(1 + \lambda_{\rm esc}\right) e^{-\lambda_{\rm esc}} \tag{4}$$

and assuming that the escaping particles have velocity v_0 equal to the thermal velocity.

d. Should H and O thermally escape on any of these planets?

e. Describe three different mechanisms of nonthermal escape.

3.* For Experienced Programmers: *Planet Occurrence Rates

In this problem, you will use data from the NASA *Kepler* mission to estimate the frequency of planets orbiting other stars.

a. Begin by writing an expression for the signal-to-noise ratio (SNR) of a single transit of a planet with radius R_p orbiting star with radius $R_\star$ and noise level σ. (This noise term encompasses both astrophysical noise and instrumental noise.)

b. Assuming that the total SNR increases with the square root of the number of transits observed and that the star was observed for $N_{\rm days}$ days, convert your answer to (a) into the cumulative SNR for multiple transits of a planet with period P days.

c. Using Kepler's third law, write the geometric likelihood of transit in terms of the orbital period P, the stellar radius $R_\star$, and the stellar mass $M_\star$.

d. Download the stellar table from the course website and compute the effective number of days $N_{\rm days}$ that *Kepler* observed each star. $N_{\rm days}$ is the product of the "dataspan" (the number of days between the first and last observation of the star) and the "duty cycle" (the fraction of time that *Kepler* observed the star) . This table is a subset of the *Kepler* stellar table containing 92,157 moderately quiet Sun-like stars that were observed for a significant fraction of the *Kepler* mission. The column headings are the same as in the original table and are described at the top of the file. For more details, see https://exoplanetarchive.ipac.caltech.edu/docs/API_keplerstellar_columns.html#stellar

e. For each star in the table, compute the duration and depth of a central transit of an Earth-sized planet with an orbital period of 10 days.

f. The "rrmscdpp[XX]p[Y]" columns provide a measurement of the noise level of the *Kepler* photometry on timescales of XX.Y hours. For instance, "rrmscdpp06p0" is the noise on 6-hour timescales and "rrmscdpp12p5" is the noise observed on 12.5-hour timescales. Use these columns to estimate the typical noise of each star over the transit timescales you found in (e). Note that the "rrmscdpp[XX]p[Y]" columns quote the noise in parts per million (ppm).

g. Plug your answers to parts (e) and (f) into the expression you found in part (b) to determine the cumulative SNR at which a $1\,\mathrm{R}_\oplus$ planet in a 10-day orbit could have been detected around each star.

h. Report the number of stars for which the cumulative SNR exceeds the nominal detection threshold of 7.1σ.

i. Determine the number of stars "searched" for Earth-sized planets on 10-day orbits by correcting your answer to part (i) by the geometric likelihood of transit you found in part (c).

j. Download the file of detected planets from the course website and restrict the table to contain only the planets orbiting stars listed in the stellar table you downloaded in part (d). The planet table is the set of planet candidates and confirmed planets detected in the Q1-Q16 Kepler data. The column headers are described at the top of the file and at https://exoplanetarchive.ipac.caltech.edu/docs/API_kepcandidate_columns.html.

k. Count the number of planets with periods within 20% of 10 days and radii within 20% of $1\,\mathrm{R}_\oplus$.

l. Combine your answers to parts (i) and (k) to estimate the frequency of Earth-sized planets with periods near 10 days orbiting Sun-like stars.

m. Compare your answer to the values found by Burke et al. (2015, ApJ, 809, 8) for small planets with longer orbital periods. (The stellar file you downloaded in part (d) was generated using nearly the same sample cuts as in Burke et al. 2015).

n. *Optional Extra Challenge:* Repeat the occurrence calculation for different regions of parameter space and produce a two-dimensional plot of planet occurrence as a function of orbital period and planet radius. Compare your findings to the exoplanet literature.

4.* Programming-Free Alternative to Problem 3: *Highlights from the Literature

Select one paper from each category and summarize the motivation, methodology, results, and conclusions. Include a representative figure from each paper and write a few sentences explaining how the figure connects to the rest of the paper. Your full answer for each paper should be 1–2 paragraphs. You may summarize up to three additional papers (6 papers total) for extra credit.

a. Pick 1: Seager & Sasselov 2000; Charbonneau et al. 2002; Knutson et al. 2007

b. Pick 1: Morley et al. 2015; Sing et al. 2016

c. Pick 1: Kaltenegger et al. 2007; Zahnle & Catling 2017