## Astronomy 7B Final May 2018

Name: _			
Section:			

There are 5 problems that are required and 1 optional bonus problem that can only improve your grade.

Write your answers on these sheets showing all of your work. Feel free to use the backs of pages as well, but please clearly label which work corresponds to which problem.

Calculators are allowed to perform arithmetic. Please turn off all cellphones.

If you have any questions while taking the midterm, get the attention of one of the GSIs or the instructor.

Budget your time; you will have from 8:10 am to 11:10 am to complete the exam. Of course, you are free to hand in your exam before 11:10 am. Make sure that you have time to at least briefly think about every required question on the midterm.

You do not need to work on the questions in order, so it is OK to skip a question and come back to it later.

## Constants

c = 
$$3.00 \times 10^{10}$$
 cm/s =  $3.00 \times 10^{8}$  m/s  
h =  $6.623 \times 10^{-27}$  erg s =  $6.623 \times 10^{-34}$  m<sup>2</sup> kg s<sup>-1</sup>  
 $\sigma = 5.67 \times 10^{-5}$  erg s<sup>-1</sup> cm<sup>-2</sup> K<sup>-4</sup> =  $5.67 \times 10^{-8}$  W m<sup>-2</sup> K<sup>-4</sup>  
 $a = 7.57 \times 10^{-15}$  erg cm<sup>-3</sup> K<sup>-4</sup> =  $7.57 \times 10^{-16}$  J m<sup>-3</sup> K<sup>-4</sup>  
 $k = 1.38 \times 10^{-16}$  erg/K =  $1.38 \times 10^{-23}$  J/K  
 $G = 6.67 \times 10^{-8}$  dyne cm<sup>2</sup>/g<sup>2</sup> =  $6.67 \times 10^{-11}$  m<sup>3</sup> kg<sup>-1</sup> s<sup>-2</sup>  
1 dyne = 1 g cm/s<sup>2</sup> =  $10^{-5}$  N  
 $m_p = 1.673 \times 10^{-24}$  g =  $1.673 \times 10^{-27}$  kg  
 $m_n = 1.675 \times 10^{-24}$  g =  $1.675 \times 10^{-27}$  kg  
 $m_e = 9.11 \times 10^{-28}$  g =  $9.11 \times 10^{-31}$  kg  
 $\sigma_{\text{Thomson}} = 6 \times 10^{-25}$  cm<sup>2</sup> =  $6 \times 10^{-29}$  m<sup>2</sup>  
 $M_{\oplus} = 6 \times 10^{27}$  g =  $6 \times 10^{24}$  kg  
 $R_{\oplus} = 6.4 \times 10^{8}$  cm =  $6.4 \times 10^{6}$  m =  $6.4 \times 10^{3}$  km  
 $L_{\odot} = 3.90 \times 10^{33}$  erg/s =  $3.90 \times 10^{26}$  W  
 $M_{\odot} = 2.0 \times 10^{33}$  g =  $2.0 \times 10^{30}$  kg  
 $R_{\odot} = 7.0 \times 10^{10}$  cm =  $7.0 \times 10^{8}$  m  
 $T_{\odot} \approx 5800$  K (surface temperature)  
1 AU =  $1.5 \times 10^{13}$  cm =  $1.5 \times 10^{11}$  m  
1 pc =  $3.09 \times 10^{18}$  cm =  $3.09 \times 10^{16}$  m  
1 eV =  $1.6 \times 10^{-12}$  erg =  $1.6 \times 10^{-19}$  J  
1 radian =  $206265$  arcsec  
1 Angstrom =  $10^{-8}$  cm =  $10^{-10}$  m  
1 year = 12 months =  $365$  days =  $3.15 \times 10^{7}$  s

## Some Useful Formulae

Kepler's Third Law:

$$P = \frac{2\pi}{\sqrt{G(m_1 + m_2)}} a^{3/2}$$

Equation for an ellipse:

$$r = \frac{a(1 - e^2)}{1 + e\cos\theta}$$

Orbital energy and angular momentum in the center-of-mass frame:

$$E = -\frac{Gm_1m_2}{2a}$$

$$L = \frac{m_1 m_2}{m_1 + m_2} \sqrt{G(m_1 + m_2)a(1 - e^2)}$$

Wien Peak Law for Planck function  $B_{\lambda}$ :

$$h\nu_{\rm peak} = hc/\lambda_{\rm peak} \approx 5kT$$

For non-spinning black holes:

$$R_{\rm ISCO} = 6R_g$$
 and  $R_{\rm EH} = 2R_g$ 

For maximally spinning, prograde black holes:

$$R_{\rm ISCO} = 1R_q$$
 and  $R_{\rm EH} = 1R_q$ 

Friedmann Equation:

$$(\dot{a}/a)^2 - 8\pi Gu/(3c^2) = -c.c./a^2$$

Redshift:

$$z \equiv (\lambda_o - \lambda_e)/\lambda_e$$