Astro 7B – Problem Set 5

1 Reverberation Mapping

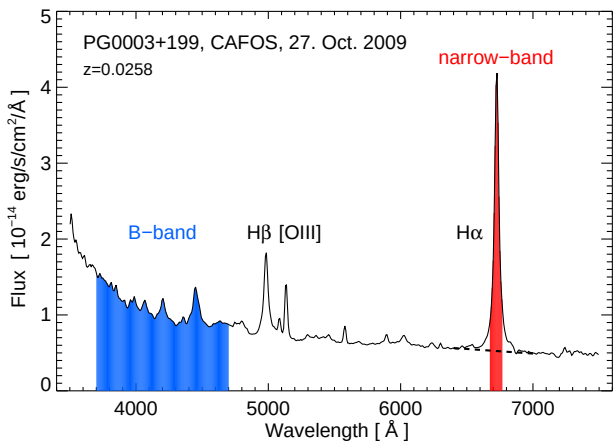

Figure 1: Spectrum of the active galactic nucleus PG0003+199 (Haas et al. 2011).

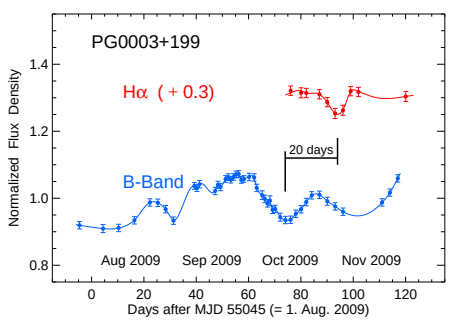

Figure 2: Two light curves of PG0003+199 in two different bandpasses.

Figures 1 and 2 were shown in class: they display the optical spectrum of the active galactic nucleus PG0003+199, and two light curves taken in the H-α bandpass and the broadband B-filter.

Estimate using this data the central black hole mass of PG0003+199 (express in $M_\odot$). When measuring the width of the H-α emission line, take the full-width-at-half-maximum (FWHM). Assume the accretion disk around the black hole is viewed edge-on ($i = 90$ deg).

2 Apparent Superluminal Motion

In class we derived the formula for the apparent transverse velocity of a jet traveling at speed v, seen at an angle θ relative to the jet axis:

$$v_{\rm app} = \frac{v \sin\theta}{1 - (v/c)\cos\theta} \tag{1}$$

In real life, the jet (i.e., the magnetized plasma blobs in the jet) must travel at $v < c$. Equivalently, $\beta \equiv v/c < 1$.

(a) Make a plot of $\beta_{\rm app} = v_{\rm app}/c$ as a function of θ between 0 and 90 degrees, assuming $\beta = 0.5$. Overlay on this plot the results for $\beta_{\rm app}(\theta)$ for $\beta = 0.9$. Does θ need to be very small for the observer to perceive faster-than-light motion?

(b) Derive an expression for $\theta = \theta_{\rm max}(\beta)$ such that $\beta_{\rm app}$ is maximized.

3 More Reverberation Mapping: Lag Times for Arbitrary Disk Inclination

In class we showed that the average lag time

$$\overline{\Delta t_{\rm lag}} = r/c \tag{2}$$

for disks viewed either face-on (inclination $i = 0$) or edge-on (inclination $i = 90$ deg). Here r is the radius of the gas ring which becomes ionized by UV photons and which subsequently emits Hα photons, and c is the speed of light.

These Hα photons are received by the observer a time $\Delta t_{\rm lag}$ after the original UV photons are received. In general $\Delta t_{\rm lag}$ depends on location in the gas ring. In class we considered the 4 "compass points" on the ring: "North", "South", "East", and "West". In the equation above, the overbar denotes an AVERAGE over these 4 compass points.

(a) **Derive an expression for the average lag time $\overline{\Delta t_{\rm lag}}$ (averaged over these same 4 compass points) for a disk viewed at arbitrary inclination i. Show all your work and reasoning carefully.**

(b) **Finally write down a formula for the mass of the black hole in terms of $\Delta\lambda/\lambda$, i, $\overline{\Delta t_{\rm lag}}$, and fundamental constants.** The notation is the same as that used in class, and you are free to just use whatever lecture notes you have, but please show all your work and

reasoning carefully. In particular, be sure to think about how the measured $\Delta\lambda/\lambda$ relates to the disk inclination i. Your answer for part (b) should have a dependence on i.