Order-of-Magnitude Physics: Hand-Waving as Performance Art

Physics is physical. It is one thing to be told that Tyrannosaurus Rex is 20 feet tall, and another to envision—as Feynman's father had the young Feynman imagine—the giant lizard's head smash through a second-story window. When a physical system is understood, it becomes embedded in our sense memory. This is why we do astrophysics: to get the physics in our blood, to develop a feeling for the subject, to expand our intuition to encompass the universe.

How can we access the tactile and visceral in astrophysics? Live demonstrations, images, and films are obvious and effective tools. Facts stick and cohere when what's up there is connected to what's down here: when convection in stars is identified with hot miso soup. As for pretty pictures, astronomy has an inexhaustible supply.[1] And if a picture can convey a thousand words, what more an animation? Movie clips of the N-body problem can communicate in minutes what blackboards strung out over sleepy hours cannot.

But jpegs, mpegs, and analogies do not alone make a physics education. They motivate, inspire, and keep the class awake, but they are the spices of lecture and cannot provide a full meal. At the end of the day, we need the math. The problem is that there's a lot of math, and not only is much of it tedious, but if not presented carefully, it can obscure the physics. Students can perform long technical calculations whose meaning they cannot articulate. Even monkeys can do algebra.[2] Yes, on the one hand, long derivations lie at the foundation of physics, and by extension of technological civilization. But unless the derivation is itself instructive, it is better left for the privacy of the dorm. In the $\sim$50 minutes allotted to the instructor to win hearts and minds, collecting terms on the right-hand side and keeping track of 2's does not constitute quality time.

The antidote to monkeying around with math is order-of-magnitude (OOM) physics: the art of estimating any quantity under the Sun, and beyond, to within a factor of 10. To do OOM physics is to distill—to make insightful, simplifying assumptions—to discard the dozen terms in the equation that don't matter and keep the two that do—in short, to do physics. In OOM physics, scalings matter more than coefficients. Would it kill anyone to drop the π? Although mnemonics, heuristic arguments, and semi-classical treatments are criticized as dirty tricks, these devices should also be recognized as friendly introductory guides. We have to start somewhere, and what better way to start than with ideas that build intuition and enable us to crank out real numbers in real time—on blackboards and backs of envelopes.

It is hard to overestimate the empowerment that order-of-magnitude estimation brings. I spent years in physics feeling embarrassed that I was not able to calculate anything

[1] See the "Astronomy Picture of the Day": `http://apod.nasa.gov/apod/astropix.html`.

[2] "Basic mathematical rules are encoded by primate prefrontal cortex neurons." Bongard & Nieder 2010, *PNAS*, 107, 2277.

"useful." Fortunately certain mentors encouraged me (and it is almost always human contact that makes the difference, not books or computers) to close the textbook and stop coding, and instead attack problems armed only with personal experience and grade-school arithmetic. It sounds like anti-science: using "common sense" to address seemingly uncommon problems. Yet pre-college math and science, plus a willingness to "make it up as you go along," are the only tools required for estimating, e.g., the spill rate of the 2010 BP oil disaster in the Gulf of Mexico. This is a calculation that industry executives could not (or refused to) get right for weeks, but which can be made in minutes by watching a video of the broken pipe and scribbling some numbers.[3] The relevant concepts are in Lecture 1 of my OOM physics class,[4] which enable one to cost-analyze Obama's inauguration; to compare the mass in plastic to phytoplankton in the Pacific; and to assess whether solar panels on the roofs of campus buildings can supply our energy needs.[5]

Terrestrial "everyday" physics is the perfect vehicle for learning OOM physics because answers can be checked by direct experimentation ("what is the yield stress of everyday solids?" $\Rightarrow$ "what is the largest carrot you can pull apart with your hands?"). After practicing on Earth, we can make the leap to outer space ("how massive must objects be before gravity makes them round?").

Even forefront research can begin (and end!) on the back of a cocktail napkin. The utility of OOM estimation extends through all phases of research. At a project's outset, evaluating a few numbers can decide whether a speculation holds water or whether it should be abandoned. Later, in the thick of a complex experiment, OOM estimation helps to diagnose. And at a project's close, OOM physics provides the gift-wrap: the most compelling discoveries are not only supported by reams of data, but can also be communicated through a toy model or a few lines of algebra.

In OOM physics, it's not so much about getting it right, so much as trying. In love and qualifying exams, it is better to have estimated and erred than never to have estimated at all. If you're wrong you won't forget it, and will better appreciate nature's subtlety.

Order-of-magnitude estimation equips us to become not only better scientists but also better citizens. The numbers bandied by newspapers and politicians—from "hundreds of tons" of CO_2 to "trillion-dollar" stimulus packages—are intended to shock-awe-and-sell but are meaningless if not put in context. In physics, "big" or "small" is defined by comparison. How much sounder would our public policy be if we had an objective basis for deciding what is "minor" and what is "major" by doing a few sums, on paper or in our heads.

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[3] For a history of events surrounding this calculation, see `http://astro.berkeley.edu/~echiang/bp/bp.html`

[4] See `http://astro.berkeley.edu/~echiang/oom/oom.html`

[5] We also review Fermi's eponymous problem, "How many piano tuners are in Chicago?" The answer is easy: who cares. The only problems worth solving are the ones we are motivated to solve.