Fig. 7. Accretion rates for plain Bondi–Hoyle–Lyttleton flow. The crossing time corresponds to $\zeta_{HL}$. In the remainder of the section, I shall cite places in the literature where further simulations of Bondi–Hoyle–Lyttleton flow may be found.

3.2 Examples in the Literature

Hunt computed numerical solutions of Bondi–Hoyle–Lyttleton flow in two papers written in 1971 and 1979. The accretion rate suggested by equation 32 agreed well with that observed, despite the flow pattern being rather different. Hunt studied flows which were not very supersonic and were non-isothermal. Upstream of the shock, the flow pattern was very close to the original ballistic approximation. Downstream, the gas flowed almost radially towards the point mass. A summary of early calculations of Bondi–Hoyle–Lyttleton flow may be found in Shima et al. (1985). The calculations in this paper are in broad agreement with earlier work, but some differences are noted and attributed to resolution differences.

More recently, a series of calculations in three dimensions have been performed by Ruffert in a series of papers (Ruffert, 1994a; Ruffert and Arnett, 1994; Ruffert, 1994b, 1995, 1996). This series of papers used a code based on nested grids, to permit high resolution at minimal computational cost. Ruffert (1994a) details the code, and presents simulations of Bondi accretion (where the accretor is stationary). Bondi–Hoyle–Lyttleton flow was considered in Ruffert and Arnett (1994). The flow of gas with $M = 3$ and $\gamma = 5/3$ past an accretor of varying sizes ($0.01 < r/\zeta_{BH} < 10$) was studied. For accretors substantially smaller than $\zeta_{BH}$, the accretion rates obtained were in broad agreement with theoretical predictions. The flow was found to have quiescent and active phases, with smaller accretors giving larger fluctuations. However, 

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**Fig. 7.** Accretion rates for plain Bondi–Hoyle–Lyttleton flow. The crossing time corresponds to $\zeta_{HL}$. 

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Edgar 2004

Ohsugi 2018
T_{\text{eff}} \approx 1300 \text{K}

1 \text{ bar “surface” of planet}

\text{Roche lobe radius}

\text{Sonic point}

\text{Pressure balance with stellar wind}

\text{Photoionization base (} \tau_{\text{uv}} = 1)\n
\begin{align*}
6 R_p & \quad \text{P } \sim 10 \text{ picobar} \\
4.5 R_p & \quad T_{\text{wind}} \approx 3000-10000 \text{ K} \\
2-4 R_p & \\
1.1 R_p & \quad P \sim \text{nanobar} \sim g/\kappa \sim g/(\sigma_{bf}/\mu m_H) \\
R_p \sim 10^{10} \text{ cm} & \quad T_{\text{eff}} \approx 1300 \text{K} \end{align*}

Murray-Clay, EC, & Murray 09
A disintegrating Super-Moon

- Opacity must be due to grains
- Coriolis force + stellar radiation pressure creates trailing tail
- Tail causes prolonged egress
- Scattered light off head of “comet” causes pre-ingress bump

Rappaport, Levine, EC+ 12
Hydrodynamic model (Perez-Becker + EC 13)
(1D) Mass, momentum, and energy conservation

\[
\frac{\partial}{\partial r} (r^2 \rho v) = 0
\]

\[
\rho v \frac{\partial v}{\partial r} = -\frac{\partial P}{\partial r} - \frac{GM_p \rho}{r^2} + \frac{3GM_\star \rho r}{\alpha^3}
\]

- pressure gradient force
- gravitational attraction from planet
- centrifugal force & attraction from star

\[
\rho v \frac{\partial}{\partial r} \left[ \frac{kT}{(\gamma - 1) \mu} \right] = \frac{kTv \partial \rho}{\mu} \frac{\partial \rho}{\partial r} + \Gamma
\]

- change in internal thermal energy
- PdV work cooling
- heating from dust-gas collisions & latent heat from condensing grains
Little orbital evolution from mass loss; in-situ formation possible
Catastrophic phase 1/1000 of lifetime
Observed in 1/150,000 stars: ~1/25 of stars could have a close-in super-Mercury
Photoevaporation of planet atmospheres and the creation of the “Fulton gap”

Figure 4. Left: distribution of planet radii and orbital periods. Right: same as left but with insolation flux relative to Earth on the horizontal axis. In both plots, an underdensity of points appears between 1.5 and 2.0 $R_\oplus$. 

Trends with Host-Star Mass

We plot planet size vs. stellar mass in Figure 8 in order to investigate potential changes in the structure of the planet radius distribution as a function of stellar host mass. This Figure shows that the transition radius between the two populations increases monotonically with stellar mass. The gap occurs near 1.6 $R_\oplus$ for planets orbiting host stars with masses near 0.8 $M_\odot$ and moves to $\sim$2.0 $R_\oplus$ for planets orbiting stars with masses above 1.2 $M_\odot$. We also split the sample into three bins of stellar mass: $M_\star$ < 0.96 $M_\odot$, $M_\star$ = 0.96–1.11 $M_\odot$, and $M_\star$ > 1.11 $M_\odot$. We chose bin boundaries such that the three bins captured equal numbers of planets. Figure 9 shows the planet population in the $P$-$R_p$ and $S_{inc}$-$R_p$ planes for each of the three mass bins. The gap is clearly visible in each of the three stellar mass bins, and appears wider than the gap from the combined sample shown in Figure 6.

We observe several trends with stellar mass. First, the typical size of super-Earth and sub-Neptune planets increases with increasing stellar mass, an observation that we quantify later in this section. This explains why the planet populations are better separated when one considers a narrow range of stellar mass; when all three mass groups are combined the distributions overlap. It also helps to clarify why the planet populations in Van Eylen et al. (2017) seemed to be more separated compared to those in F17. The asteroseismic sample was heavily weighted toward stars more massive than the sun, and is more directly comparable to the $P$–$R_p$ distribution of our high mass bin. The top right panel of our Figure 9 is a closer match to Figure 2 from Van Eylen et al. (2017) than the upper left panel of our Figure 9.

To quantify the change in typical planet size with stellar mass, we calculated the mean planet radius for sub-Neptunes (1.7–4.0 $R_\oplus$, and $P<$ 100 days) and super-Earths (1.0–1.7 $R_\oplus$, and $P<$ 30 days). We weighted each radius by the weights used in the occurrence calculations, described in Section 4.2. Figure 10 shows these mean planet parameters as a function of stellar mass. Consistent with visual inspection of Figure 9, we see monotonically increasing planet size with increasing stellar host star mass in both the super-Earth and sub-Neptune planets.

Although the trend with stellar mass is strong, we caution that stellar metallicity may be a confounding factor. More massive stars are younger on average and are thus more metal-rich due to galactic chemical enrichment. Indeed, Petigura et al. (2018) observed a correlation between planet size and host star metallicity in the CKS sample and this correlation has been observed previously in many different samples (e.g. Santos et al. 2004; Fischer & Valenti 2005; Sousa et al. 2008; Ghezzi et al. 2010; Buchhave et al. 2014; Schlaufman 2015; Wang & Fischer 2015). The solid component of the protoplanetary disk likely tracks both stellar metallicity and stellar mass. Therefore, we expect planet size to be correlated with both stellar mass and metallicity. Future studies spanning a larger range of stellar mass and metallicity are necessary to resolve this ambiguity.

Previous studies have noted a desert of highly-irradiated sub-Neptune planets (see, e.g., Lundkvist et al. 2016 and Mazeh et al. 2016). We observe this sub-Neptune desert in our three mass bins (Figure 9), but note that it shifts to higher incident stellar flux around high mass stars. This trend is highly significant.
Galactic magnetic field (M51)

**B-magnitude in microG**

B-direction from linearly polarized radio emission

Field strength estimated by assuming energy equipartition with non-thermal synchrotron-emitting electrons

Fletcher+11
We suppose that the field at the coronal base has an energy density greater than the thermal energy density, so that an initially subsonic flow will follow the field-lines. Gas starting at sufficiently low latitudes will reach the equator at points not too far from the star, where the magnetic energy density is still larger than the thermal. Even if there were no hot gas outside the region defined by the loop $ABA'$ in Fig. 1, exerting an inward pressure, the gas within $ABA'$ would reach equilibrium: a very slight denting of the field-lines would generate the discontinuity in the magnetic pressure $H^2/8\pi$ that would balance the discontinuity in thermal pressure. But gas expanding along field lines such as $EC$ cannot reach such a state of hydrostatic equilibrium. Before it has expanded far enough to reach the equator, it will find that its pressure exceeds the magnetic pressure, so that it will cease to flow along prescribed, nearly dipole field-lines: instead it will expand more-or-less radially, dragging the field with it.

The picture we arrive at finally is as in Fig. 1. There is a dead zone (1) in which the closed, approximately dipolar field-loops hold in the gas and keep it rotating with the star’s angular velocity $\Omega$. The density field $\rho$ along each field-line is given by the component of hydrostatic support along the field: assuming isothermality with sound speed $a$. 

\[ \frac{dp}{ds} = \frac{dp}{ds} \quad \text{assuming isothermality with sound speed $a$.} \]
“Pulsar wind nebula” in Crab Nebula
MHD model

- **Acceleration**
  - Magneto-centrifugal force (Blandford-Payne 1982)
    - Like a force worked a bead when swing a wire with a bead
  - Magnetic pressure force
    - Like a force when stretch a spring
    - Direct extract a energy from a rotating black hole
- **Collimation**
  - Magnetic pinch (hoop stress)
    - Like a force when the shrink a rubber band

![Diagram of MHD model](image)

**Accretion onto a black hole**

- Time = 0 $R_g/c$
- Rest mass density $\rho$
- Lorentz factor $\gamma$
- HAMR

Hesp, Liska, Tchekhovskoy
Uniform vertical background seed field with plasma $\beta=1000$

“channel solution”
MRI accretion

- Turbulence and transport are consequences of differential rotation and magnetism.

- The MRI is an effective dynamo: amplifies B and even produces magnetic cycles (like on the Sun).

- The flow is *turbulent*, not viscous. Turbulence is a property of the flow; viscosity is a property of the fluid.

- An MRI-turbulent disk and a viscous accretion disk having the same total alpha behave differently, especially in 3D.
3-D

Colors denote log density

Initially poloidal field
Uniform vertical background seed field with plasma $\beta=400$

**Fig. 7.** The Maxwell and Reynolds stresses in the fiducial run Z4 compared with the Reynolds stress seen in a purely hydrodynamical simulation that is initialized with data from model Z4 at time $t = 7.5$. Without magnetic fields, the net Reynolds stress vanishes within one orbit. The time series are boxcar smoothed on a timescale of 0.25 orbits.
solid Maxwell / dashed Reynolds

uniform net vertical $B_0$
MRI as dynamo: Generation of enormous and cyclical toroidal $B_\varphi$
$t_{\text{cool}} < t_{\text{shear}} \sim t_{\text{orbit}}$

$Q \sim 1$ but fast cooling: gravitational collapse
$t_{\text{cool}} > t_{\text{shear}} \sim t_{\text{orbit}}$

$Q \sim 1$ but long cooling: “gravito-turbulent”
How do disks accrete?

Gravitational instability (GI)

Self-gravitating disks swing amplify perturbations into trailing spirals

\[ Q = \frac{c_s \kappa}{\pi G \Sigma} \sim 1 \]

\[ \frac{M_{\text{disk}}}{M_*} \sim \frac{h}{r} \]

Goldreich & Lynden-Bell 63

Muto+ 12

Shi & EC 14
\[ \alpha \sim \frac{1}{\Omega t_{cool}} \]  

Gammie 01

\[ N > |z| \] / N

![Graph and Diagram](image)
Protoplanetary disk accretion by surface layer magnetic winds

Bai & Stone 13, Bai 13

Magnetized Disk Wind

unsteady outflows?

Cosmic rays

X-rays, FUV

Fully turbulent due to the MRI

Hall effect important, magnetic polarity dependent

Layered accretion with ambipolar damping

~0.3 AU

~15-30 AU

Protoplanetary disk accretion by surface layer magnetic winds
Mantle convection

Neutrino-driven convection in supernovae

Stellar convection

The Solar Surface

- Convection (boiling water)
  - Hot gas rises (floats up) -> Brighter
  - Cool gas sinks (pulled down by gravity) -> Darker

~ 1000 km
Necessary criterion for K-H instability in Cartesian shear flow:

\[ \text{Richardson } Ri \equiv \frac{\omega_{\text{Brunt}}^2}{(\partial v/\partial z)^2} < Ri_{\text{crit}} = \frac{1}{4} \]

(see Shu for heuristic derivation)

**Kelvin-Helmholtz (KH) Instability:**
Cartesian shear, if too strong, can overturn an otherwise stably stratified atmosphere (for formal linear analysis, including analysis of contact discontinuity in \( \rho \) and \( v \), see Chandrasekhar 61)

Tremblin & EC 12
Turbulent Cascade

*Big whorls have little whorls, which feed on their velocity. Little whorls have lesser whorls, and so on to viscosity.*

Lewis Fry Richardson (cf. Jonathan Swift)

\[ \frac{\partial \varepsilon}{\partial \ell} \]

“inertial range”

\[ \varepsilon \]

“outer scale”

= energy injection

\[ \ell_{\text{inner}} \]

“inner scale”

= energy dissipation (energy goes into heat)