Problem 1. Inner Disk Edge

A gas disk orbits an object of mass $M$. Disk self-gravity is negligible. The inner edge of the disk is located at radius $a$. Gas there has a sound speed $c_s$ and an orbital angular frequency $\Omega$.

The inner edge is not perfectly sharp. At the inner edge, the gas pressure decreases (going inward) over a radial length scale $\delta$. Make an order-of-magnitude estimate for the smallest $\delta$ can be before gas at the disk edge becomes Rayleigh-unstable.

Problem 2. Clumping Sound Waves

Shu Problem Set 3, problem 1abc.

NB: The previous generation of 202 students complained that they did not get much out of Shu’s problem sets because they were excessively technical. I have considered this complaint, but still feel that certain problems are worthwhile. This one is worthwhile because it offers a situation to study gravitational instability without having to pull the Jeans swindle, and it also provides practice in deriving dispersion relations.

Problem 3. Magnetospheres

(a) Estimate the radius $r_A$ of the Earth’s magnetosphere, in units of the Earth’s radius.

(b) Estimate the radius $r_A$ of an accreting T Tauri star’s magnetosphere, assuming $\delta \sim r$ (see lecture notes).

Use $M_* \sim 1M_\odot$, $R_* \sim 3R_\odot$, a dipole field of surface strength $B_* \sim 1$ kG, and the median measured accretion rate of $\dot{M} \sim 10^{-8}M_\odot$ yr$^{-1}$.

Hot Jupiters are speculated to be deposited at $r_J \approx 0.6r_A$ (at this location, the hot Jupiter is in 2:1 resonance with the disk edge at $r_A$). Compare your estimate for $r_J$ with the observed radius for a hot Jupiter orbit (0.05 AU)$^1$.

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$^1$Given the crudeness of the theory, this comparison is not expected to yield better than order-of-magnitude agreement.