Problem 1. LOST

Desmond turns the key and your plane splits in half at an altitude of 32,000 feet above the Pacific. You are not wearing your seatbelt and go flying out.

(a) [5 points] About how long does it take for you to hit the water’s surface? Credit awarded for detailed physical arguments.

(b) [5 points] About how deep do you penetrate the water—before your velocity in the water decreases to, say, < 1 m/s? Credit awarded for detailed physical arguments.

Problem 2. Particle Drift in Protoplanetary Disks

Planets form in disks of gas and dust surrounding young stars.

Consider the young Sun, encircled by a disk of gas of surface density (mass per unit face-on area) $\Sigma \approx 10^3 \text{ g cm}^{-2}$ in the vicinity of 1 AU. The temperature of disk gas is $T \approx T_0 (a/\text{AU})^{-q}$, where $T_0 \approx 200 \text{ K}$, $a$ is the disk radius, and $q \approx 1/2$. Call the corresponding sound speed $c_s \approx \sqrt{kT/\mu}$, where $k$ is Boltzmann’s constant and $\mu$ is the mean molecular weight.

(a) [5 points] Consider a dust particle of size $s = 1 \mu\text{m}$ and internal density $\rho_p \approx 1 \text{ g cm}^{-3}$ located one scale height above the disk midplane, $z = h$ (see Problem Set 1). Estimate the time $t_{1/2}$ the particle takes to settle vertically from $z = h$ to $z = h/2$. This time is called a characteristic settling time for the dust. Provide both a symbolic answer and a numerical answer in years.

(b) [5 points] Does disk gas rotate at sub-Keplerian or super-Keplerian speeds? Estimate the difference $\Delta v$ between the gas velocity and the circular Keplerian speed $v_K \equiv \Omega a \equiv \sqrt{GM_\odot/a}$, at the midplane $z = 0$ at $a = 1$ AU. Express $\Delta v$ symbolically in terms of $c_s$, $\Omega$, $\omega$, and whatever dimensionless numbers you need. Also provide a numerical estimate.

(c) [5 points] Particles (rocks) are considered “well-entrained” in disk gas—i.e., they rotate with nearly the same velocity as disk gas—if their momentum stopping time

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1One of the better shows of the 21st century. Unlike most people, I actually liked the ending.
is short compared to the *dynamical time* $t_{\text{dyn}} \sim 1/\Omega$ [this latter time is the time for gas to change its velocity by $\mathcal{O}(1)$ (an order-unity factor)—in the case of gas in rotation, the time it takes for the disk velocity to change its direction by $\mathcal{O}(1)$ radian]. Here $m$ is the particle mass, $v_{\text{rel}}$ is the relative velocity between a particle and disk gas, and $F_D$ is the drag force on the particle.

Estimate the critical size $s_{\text{crit}}$ for which $t_{\text{stop}} \sim t_{\text{dyn}}$ at the midplane $z = 0$ at $a = 1$ AU. A numerical answer suffices. Are bodies larger or smaller than $s_{\text{crit}}$ well-entrained?

(d) [5 points] About how long does it take a critically sized particle to drift *radially* inwards due to drag? The particle starts at $z = 0$ at $a = 1$ AU, and we want to know how long it takes to reach, say, $a = 0.5$ AU. A numerical answer suffices.