Astrophysical Fluid Dynamics – Problem Set 5

Readings: Movies: Drag III, IV; skim pages 11 through 21 of the Course Reader

Note: Do not hesitate to make simplifying assumptions and rough estimates for the problems below. But please justify your assumptions whenever possible.

Problem 1. LOST

Desmond turns the key and your plane splits in half at an altitude of 32,000 feet above the Pacific. You are not wearing your seatbelt and go flying out.

(a) About how long does it take for you to hit the water's surface? Credit awarded for detailed physical arguments.

(b) About how deep do you penetrate the water? Credit awarded for detailed physical arguments.

Problem 2. Particle Drift in Protoplanetary Disks

Planets form in disks of gas and dust surrounding young stars.

Consider the young Sun, encircled by a disk of gas of surface density (mass per unit face-on area) $\Sigma \approx 10^3\,\mathrm{g\,cm^{-2}}$ in the vicinity of 1 AU. The temperature of disk gas is $T \approx T_0(a/\mathrm{AU})^{-q}$, where $T_0 \approx 200\,\mathrm{K}$, a is the disk radius, and $q \approx 1/2$. Call the corresponding sound speed $c_s \approx \sqrt{kT/\mu}$, where k is Boltzmann's constant and μ is the mean molecular weight.

(a) Consider a dust particle of size $s = 1\mu$m and internal density $\rho_p \approx 1\,\mathrm{g\,cm^{-3}}$ located one scale height above the disk midplane, $z = h$ (see problem set 1). Estimate the time the particle takes to settle *vertically* to the midplane. Provide both a symbolic answer and a numerical answer in years.

(b) Does disk gas rotate at sub-Keplerian or super-Keplerian speeds? Estimate the difference Δv between the gas velocity and the circular Keplerian speed $v_K \equiv \Omega a \equiv \sqrt{GM_\odot/a}$, at the midplane $z = 0$ at $a = 1$ AU. Express Δv symbolically in terms of c_s, Ω, a, and whatever dimensionless numbers you need. Also provide a numerical estimate.

(c) Particles (rocks) are considered well-entrained in disk gas—i.e., they rotate with nearly the same velocity as disk gas—if their *momentum stopping time*

$$t_{\mathrm{stop}} \equiv \frac{m v_{\mathrm{rel}}}{F_D} \tag{1}$$

is short compared to the dynamical time $t_{\rm dyn} \sim 1/\Omega$ [this latter time is the time for disk gas to *turn* / change its velocity (direction) by $\sim$1 radian]. Here m is the particle mass, $v_{\rm rel}$ is the relative velocity between a particle and disk gas, and F_D is the drag force on the particle.

Estimate the critical size $s_{\rm crit}$ for which $t_{\rm stop} \sim t_{\rm dyn}$ at the midplane $z = 0$ at $a = 1$ AU. A numerical answer suffices. Are bodies larger or smaller than $s_{\rm crit}$ well-entrained?

(d) About how long does it take a critically sized particle to drift *radially* inwards due to drag? The particle starts at $z = 0$ at $a = 1$ AU, and we want to know how long it takes to reach, say, $a = 0.5$ AU. A numerical answer suffices.

Problem 3. Adiabatic vs. Isothermal Sound Waves in the Classroom

Do sound waves in the classroom behave adiabatically or isothermally? Present a quantitative, order-of-magnitude proof, considering cooling by conduction alone. Recall that conduction involves the random walk of molecules. No more than a few lines of rough but still quantitative reasoning are required to solve this problem.

Audible sound waves have frequencies ν between 20 and 20000 Hz.