Astrophysical Fluid Dynamics – Problem Set 7

Readings: Shu 210–217, 230–240 (but remember error on page 237); however much of Blandford and Thorne Chapter 16 (available on-line) you like

Problem 1. Post-Shock is Sub-Sonic

Shu Problem 4a, pages 449–450. We showed in class that post-shock material flows subsonically for strong shocks. This problem shows that the result holds true for any shock, weak or strong.

Problem 2. Blowing Bubbles

A star emits a wind of constant velocity v_w and mass-loss rate $\dot{M}$. The wind impacts the surrounding interstellar medium of density ρ_0 and terminates in a shock (read: heliopause, or for the non-solar case, astrosphere).[1]

(a) Argue your way to an expression for the radius R of the shock as a function of time t, assuming that all of the mechanical energy of the wind is used to drive outwards the shocked (swept-up) ISM.

(b) What is the velocity of the flow just behind the shock, evaluated in the star's rest frame (not following the shock)? Assume a strong shock and $\gamma = 5/3$.

How can you reconcile this result with the fact that far behind the shock the flow velocity must be outward at speed v_w? Thereby deduce that there are actually two shocks—a "forward" shock whose radius R you have solved for in (a), and a "reverse" shock whose radius $R_1 < R$.[2] In the forward shock, the ISM gets shocked. In the reverse shock, the wind gets shocked.

Is the velocity you have just solved for appropriate for shocked ISM or shocked wind?

(c) The star, which has been blowing a wind for a long time, goes supernova. Argue your way to decide how the radius R_s of the supernova blast wave scales with time t, assuming that all of the mechanical energy of the supernova is used to drive outwards the shocked (swept-up) stellar wind. Just the scaling for this part is sufficient; no need to supply the coefficient.

Problem 3. Bow Shock

Consider two stars of masses M_1 and M_2 in a circular binary orbit. Star 1 emits a wind of $\dot{M}_1$ and constant speed v_1, and likewise for star 2. The two winds will collide in what is called a "bow shock". The surface of the bow shock is defined by the condition that the momentum carried by

[1] See Wood et al. 2002, 2005 for impressive measurements of the astrospheres of other stars.

[2] See the classic papers on this subject by Castor, McCray, and Weaver 1975 and Weaver et al. 1977.

star 1's wind, measured perpendicular to the bow shock and across the bow shock, balances the momentum carried by star 2's wind, also measured perpendicular to the bow shock and across the bow shock:

$$(\rho_1 v_{1,\perp}) \times v_{1,\perp} = (\rho_2 v_{2,\perp}) \times v_{2,\perp} \tag{1}$$

Here we are ignoring the pressure of each wind (such winds are called "cold" flows), which is a fine approximation for O star winds.

As with the previous problem, what we are calling a single bow shock actually comprises 2 shocks—wind 1 gets shocked on the side of the bow shock closer to star 1, while wind 2 gets shocked on the side of the bow shock closer to star 2. For more details, see, e.g., Luo, McCray, and MacLow 1990, or Stevens, Blondin, & Pollack 1992.

(a) Choose a frame rotating with the binary so that star 1 is fixed at the origin and star 2 is located a distance D away on the x-axis. Within what distance of the origin can we ignore rotational forces? That is, within what distance of the binary can we approximate an individual wind as being a purely radial flow directed away from a star?

Give an approximate analytic expression for this distance for the simple case $v_1 = v_2$ and $M_1 = M_2$. Also supply a numerical estimate for $v_1 = v_2 = 2000\,\mathrm{km\,s^{-1}}$, $M_1 = M_2 = 10M_\odot$, and $D = 10R_*$, where $R_* = 3R_\odot$ is the individual stellar radius. Express your numerical estimate in units of D.

(b) Ignoring rotational effects, show that the local slope of the bow shock surface obeys

$$\frac{dy}{dx} = \frac{R^{1/2} r_2 \sin\beta_1 + r_1 \sin\beta_2}{R^{1/2} r_2 \cos\beta_1 + r_1 \cos\beta_2} \tag{2}$$

where $\vec{r}_1$ is the distance vector between star 1 and a given point on the bow shock, β_1 is the angle that $\vec{r}_1$ makes with respect to the x-axis, and similarly for 2-variables. Here the constant

$$R \equiv \frac{\dot{M}_1 v_1}{\dot{M}_2 v_2} \tag{3}$$

(c) Decide the location of the bow shock on the x-axis only, for $\dot{M}_1/\dot{M}_2 = 10$ and $v_1 = v_2$, in terms of D.

(d-OPTIONAL) Draw the bow shock surface using (b) and (c).

OPTIONAL Problem 4. Magic

(a) Reproduce equation (17.18) of Shu, the transformed momentum equation.

(b) Decide whether the vanishing of the dependence on R depends on having the correct choice for the similarity variable $R(t) \propto t^{2/5}$. (Shu uses $r_{\rm sh}(t)$ instead of $R(t)$.) For example, you might try $R(t) \propto t^{\beta}$ as the similarity variable, with β arbitrary.

NB: In (17.11) and (17.12), Shu defines the dimensionless functions $v(\xi)$ and $p(\xi)$ differently than we did in class. He inserts an extra ξ into v and an extra ξ^2 into p. It is not clear why he does this. By comparison, our definitions of v and p in class are the same as those of Blandford & Thorne: they simply normalize using the velocity and pressure at the edge of the bubble. Ordinary differential equations result from either set of definitions, and the normalizations from either set of definitions are valid (i.e., $p(\xi = 1) = P_2$ and $v(\xi = 1) = U_{\rm sh} = \dot{R}$ using either set of definitions.)