Astrophysical Fluid Dynamics – Problem Set 9

Readings: Hand-outs by Sturrock; Shu pages 302–310; Shu pages 338 to the end of the first complete paragraph on page 341; however much of the papers by Weber & Davis (1967) and Blandford & Payne (1982) you care to read

Problem 1. MHD Modes

(a) Derive the eigenmodes for fast and slow magneto-sonic waves having $\vec{k}$ parallel to $\vec{B}_0 = B_0\hat{z}$. Write down the Cartesian eigenmode components for the perturbation velocity $\delta\vec{u}$ and the perturbation magnetic field $\delta\vec{B}$.

Also state whether the mode is compressive.

(b) Repeat (a), but for $\vec{k}$ perpendicular to $\vec{B}_0$.

(c) Repeat (a), but for Alfven waves with $\vec{k}$ parallel to $\vec{B}_0$.

(d) Repeat (a), but for Alfven waves with $\vec{k}$ perpendicular to $\vec{B}_0$.

Problem 2. MHD Shocks

Shu Problem Set 3, problem 3abd (omit c).

Notice that part (a) has typos. Correct them as part of your answer.

Part (b) also has typos: in the first equation, $u_1^2 v_{A1}^2$ should read $u_1 v_{A1}^2$. In the last equation, delete the tex'ism "rm".

Though we did not derive the MHD energy equation (Shu equation 25.3) in class, it should not appear mysterious. The extra terms are : $\partial(B^2/8\pi)/\partial t$, which says we are keeping track of the extra contribution to the energy density from the magnetic field, and $\nabla \cdot (1/4\pi)(\vec{B} \times \vec{u}) \times \vec{B} = \nabla \cdot (c/4\pi)\vec{E} \times \vec{B}$, which accounts for the divergence of an extra energy flux, namely the standard Poynting flux of electromagnetic energy [where $\vec{E}$ is evaluated in the ideal (dissipationless) MHD limit].

Problem 3. Magneto-Centrifugal Flinging

This problem is based on the classic paper on disk-driven outflows by Blandford & Payne (1982).

Consider a magnetized disk orbiting a central massive object.

Examine a magnetic field line rooted to the disk midplane at distance a from the central object. Above the disk, the field line is straight and tilted at an angle θ measured from the vertical (disk normal), and points away from the star. The field is mirror-symmetric about the disk plane (but naturally points toward the disk below the midplane and points away from the disk above the midplane). Take the field to be axisymmetric also.

For what angles θ will plasma slide unstably along the field, away from the star, thereby producing a magneto-centrifugal outflow? Model a field line as being a rigid immovable wire, so that a fluid parcel can travel only parallel to the field. Work in the limit that ∇P parallel to the field is negligible.

This looks like an MHD problem, but with the given simplifications, it reduces to a bead-on-a-wire mechanics problem. Remember that there are two kinds of equilibrium: stable and unstable.