Astro 250 – Order-of-Magnitude Physics – Final Exam

Closed Book and Closed Computer
Except for Purcell’s Sheet of Useful Numbers

Duration: However long you want, in one continuous sitting.

Hand in your finished exam on Monday May 18, 3:30 pm, Hearst Mining 310.

Total of 8 problems of varying length but equal weight.

“It is better to have estimated and erred than never to have estimated at all.”
Problem 1. Practice, Practice, Practice\textsuperscript{1}

The violin box is a Helmholtz resonator. The box has two f-shaped holes (“f-holes”) carved into the top, of length \(\sim 10\) cm and width \(\sim 7\) mm. The thickness of the wood is about 2 mm.

Estimate the fundamental Helmholtz frequency.

Problem 2. It’s Always Greener on the Other Side of the Fence (suggested by Edwin Kite)

A demonstration on Cal Day: A 5-gallon transparent drum is upturned over a patch of grass in bright sunshine. Estimate from first principles how long it takes for the grass to significantly reduce the CO\textsubscript{2} within the drum. CO\textsubscript{2} constitutes about 350 ppmv (parts per million by volume) of the atmosphere.

Problem 3. Let’s Do the Twist

Venus has a mass \(M\) and radius \(R\) similar to the Earth’s. Its orbit is located 0.7 AU from the Sun, it rotates retrograde with a period of 240 days, and it has neither oceans nor a moon.

(a) Estimate the timescale, \(\omega/\dot{\omega}\), over which tides raised by the Sun on Venus change the Venusian angular rotation rate \(\omega\).

(b) Assume such tides have been acting over the 4.6 Gyr age of the solar system. Estimate the initial rotation period of Venus 4.6 Gyr ago, \(2\pi/\omega_{\text{init}}\).

(c) Venus, as with all rocky planets, is thought to have formed by accreting planetesimals of mass \(m \ll M\). Assume planetesimals collide with the young Venus inelastically, and that their orbital eccentricities and inclinations prior to impact are large, on the order of unity. Further assume planetesimals to have a single size \(r\) and mass \(m\).

Estimate \(r\) such that the nascent Venus spins at the correct initial rate. Provide both a symbolic and numerical answer.

\textsuperscript{1}Reply of the concert pianist Arthur Rubinstein when asked how to get to Carnegie Hall.
Problem 4. Talking Smack

The Spartans fought the Persians at Thermopylae, a mountain pass about 100 meters wide. According to the Greek historian Herodotus, when a Spartan soldier was informed that the Persian arrows would be so numerous as “to block out the Sun,” he replied, “So much the better...then we shall fight our battle in the shade.”

Decide whether the Persian threat of arrows darkening the sky is credible.

Problem 5. Riddle of Giza

The Great Pyramid of Giza, built to entomb Egyptian pharaoh Khufu, stands about 150 meters tall. Its base is a square of length 230 m.

Estimate the number of workers required to build the Great Pyramid.

Problem 6. Fire Up the Grill

A typical barbeque lighter uses a piezoelectric crystal to create a spark. A pair of electrodes mounted on opposite sides of the crystal connect to wires whose ends are nearly (but not) touching. (The air gap between the wires is just like the gap in an automobile spark plug.) When the crystal is squeezed hard enough, a spark jumps between the wire ends.

Take the crystal to be a cube of length 1 cm, and the gap distance to be 3 mm. Estimate from first principles the amount of force one needs to apply to the crystal to create a spark.

Problem 7. Saving Private Ryan

To what minimum depth of water would a World War II frogman (combat swimmer) need to dive to avoid being killed by machine gun bullets fired from the beaches of Normandy?
**Problem 8.** Best Served Cold

A typical computer hard drive consists of a disk ("platter") of diameter 3.5 inches. Data is stored in concentric tracks (like tree rings) of magnetic patterns: 1’s and 0’s stored as patches of alternating magnetization.

Assume the hard disk has reached the maximum storage density set by the “superparamagnetic limit.” At this limit, the magnetic patches cannot be made any smaller, lest their magnetizations spontaneously flip, corrupting data.

(a) Estimate the maximum storage density in units of bits per square inch.

(b) Data are recorded and retrieved through “read/write heads.” The read/write head can be thought of as the needle for a record (LP) player, except that (1) the head does not touch the platter, and (2) data is organized into concentric tracks (not a continuous spiral as on a record). A servo motor moves the head to different radii to sample different tracks. Different azimuths (angles) are sampled by virtue of the platter’s rotation.

Does the servo motor that controls the head’s position in radius need to actively compensate for thermal expansion of the platter throughout the day? Give a quantitative figure of merit that explains why or why not.