1 Thalassaemia

As reported by Judith Tan of Singapore’s Straits Times, Dec 7 2005:

Thalassaemia is an inherited blood disorder. Those who have thalassaemia-major have a shortage of hemoglobin and need a blood transfusion every month to stay alive. Victims do not live past childhood unless they receive a bone marrow transplant.

Those who have thalassaemia-minor carry the gene but do not express it. They can pass the gene to their children but otherwise lead normal lives. The rules for passing the gene are the usual ones: if both parents have thalassaemia-minor, their children have a 25% chance of having thalassaemia-major, a 50% chance of having thalassaemia-minor and a 25% chance of not carrying the gene at all. If only 1 parent has thalassaemia-minor, offspring have a 50% chance of having thalassaemia-minor and 50% chance of not carrying the gene at all.

Worldwide, about $10^5$ children are born each year with thalassaemia-major.

What is the probability that you have thalassaemia-minor?

Let $T$ represent the condition Thalassaemia-major and $t$ represent Thalassaemia-minor. We are told that

$$10^5 \text{ children born e.y. with } T \Rightarrow 4 \times 10^5 \ t-t \text{ couples had kids,}$$

neglecting couples with one $T$ partner.

How many babies are born every year? There are about $6 \cdot 10^9$ people, and the population is growing slowly, though in part due to increasing life-spans. To keep the number constant, every person has one child (a couple has two). With an average lifespan of 70 (75 in the US these days, less in much of the world) years, we need a bit less than $10^8$ to keep the number of people constant. Round up to $10^8$ births to account for a little population growth.\(^1\) Assuming a couple can only have one baby per year, and the couples having babies in a given year are a random sample from all couples, this means that the probability $P(t-t)$ that both members of a couple have $t$ is

$$P(t-t) \approx \frac{4 \times 10^5}{10^8} = 0.004.$$  

Now, assuming couples are random pairs, this means that the probability that an individual has $t$ is

$$P(t) = \sqrt{P(t-t)} \approx 6\%.$$  

\(^1\)According to the US Census Website, this is about right.
This proportion should be constant across generations given our assumptions and the rules of genetics.

It’s probably not true that all people are at equal risk of having \( t \). In fact according to the U.K. Thalassaemia Society,\(^2\) Thalassaemia “occurs in a line extending through the Mediterranean, the Middle East, the Indian sub-continent and throughout South East Asia, in a region including Southern China, Thailand, the Malay Peninsula and many of the islands.” If we say this encompasses roughly half of the population of the world, people of the above origin have a probability of about .09 of carrying \( t \), by the same analysis as above, but with only half the total number of births. The rest of the world probably has close to zero chance of carrying \( t \).

2 Put Your Money Where Your Mouth Is

Estimate the sizes (in dollars) of the following enterprises in the United States:

- U.S. Government
- K–12 Education
- Entertainment (excluding sports)
- Sports
- Restaurant

U.S. Government

Last I heard, the U.S. recently passed 300,000,000 in population. However, not everyone works. I will assume as in Problem 1 that the population distribution with age is uniform. Roughly taking the ages 20-60 as working years and taking the lifespan to be 80, this cuts the working population by half. Next, I make note of the $96,000 position at Berkeley that has been posted somewhere in Leconte and I assume that this is a fairly good deal, so I will take $70,000 to be a typical American’s salary. And finally, we’ll take the tax rate to be roughly 30%. Multiplying the factors

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300,000,000 \text{ Citizens} \times \frac{1}{2} \times \$70,000 \times 0.3 = \text{ \$3 Trillion}
\]

Which sounds about right and is pretty close to the 2013 figure found on Wikipedia of $2.7 Trillion\(^3\).

K-12 Education

Let’s start by answering an easier, related question: how many K-12 students are there in the United States? All States make school attendance mandatory up until a certain age, and most students attend for the full 13 years. So, let’s make the simplifying assumption that all children aged 5-18 attend school. How many people is that? Well, we don’t know the exact age distribution, so let’s make another simplifying assumption and say it’s flat - that everyone in the US lives until a certain age (say, 75) and then abruptly dies. Then, we can estimate the number of schoolchildren in the country as

\[
3 \cdot 10^8 \cdot \frac{13}{75} \approx 5 \cdot 10^7.
\]

\(^2\)www.ukts.org/pages/background.htm

\(^3\)http://en.wikipedia.org/wiki/U.S._federal_budget
Ok, but what about the real question? Since we’re just trying to get an order-of-magnitude answer here, let’s assume that most of the cost of education goes to paying teacher salaries. I remember from my own schooling that a typical class size might be something like 25 students, which means we have $5 \cdot 10^7 / 25 \approx 2 \cdot 10^6$ teachers to pay. A teacher might make like 50 grand a year in wages, but they also get decent benefits and a retirement plan - the ones who work in public schools, anyway - so let’s estimate the total compensation of a teacher as 100,000 dollars per year. That translates into 200 billion dollars a year, just to pay the teachers. So as a rough estimate, let’s say that K-12 education is a [200 billion dollar] a year industry.

Does that number make sense? Well, the US GDP is about 14 trillion, so this means that we spend a few percent of our national wealth on education. Sounds about right.

What does the real data say? According to the 2007 Statistical Abstract of the United States, we really spend 511.2 billion per year on all K-12 education. Of course, it takes more to run a school district than just teachers; you also need administrators, janitors, counselors, secretaries, special needs professionals, etc. And, we neglected to include any non-labor costs like books, food, or maintenance. So we undershot a bit, but still got to within a factor of a few. If you wanted, you could have included a correction of a few hundred billion to account for other costs.

**Entertainment**

One way to approach this problem is to estimate the revenues of the key members of the entertainment industry and add them up. The problem with doing that is deciding what exactly to include under "entertainment." That could mean a lot of things, and it turns out that the entertainment sector is not dominated by a few big industries like movies or TV. Unless you were very careful, you probably undershot the answer by a factor of ten or so if you used this approach.

How else could I estimate the size? The problem is that people spend money on different things - I’m a movies and music guy myself, but someone else might prefer comic books, or video games, or magazines, or gambling. It’s too easy to forget something big, so instead of estimating industry-by-industry, I’ll work from my personal budget and then assume that, whatever they’re spending it on, other people are spending money on entertainment at about the same rate I am.

Personally, after dropping 60 dollars a month for cable + Netflix, I’d say I spend about 10 dollars a week on movies, music, books, concerts, and so forth. That’s a total of 100 dollars a month, or about 1200 dollars a year.

Is that number reasonable? I’ll try to estimate it in another way. As you all know, grad students at Berkeley make roughly 1600 dollars per month, about 1200 dollars of which we immediately fork over as rent, food, clothes, and other necessities. That leaves 400 dollars, which means that if I can indeed afford to spend 100 dollars a month on "entertainment" and still save some money. Note that this means I spend about 6 percent of my monthly income on entertainment, which sounds reasonable.

Is that number about right for an average American? Well, I make less money than average, but I also don’t have any kids to support. What’s more, I don’t really have the time to enjoy much more entertainment than I already am. Even if I had more money, I doubt I’d go to movies or concerts more often. So assuming I’m about as busy as your average American, I think it’s reasonable to extrapolate from myself here.

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4http://www.census.gov/compendia/statatab/2007/tables/07s0205.xls
So, if everyone in the United States spent money like I do, the entertainment industry would have a revenue of about \(3 \cdot 10^8 \cdot 1200 \approx 400 \text{ billion dollars.}\)

Unfortunately, while the Department of Commerce does keep data on US GDP by industry, it doesn’t break down the industries in a way that would be useful for us here. Instead, I found a website\(^5\) that gives a total value of 923.9 billion in 2008 for the "Entertainment and Media" industry. This figure includes things like advertising on FM radio that I didn’t even begin to take into account here, and anyway, a factor of two isn’t bad.

**Sports**

Lots of people calculated the revenue of a typical major spectator sport league (NBA, NFL, etc.) and got an answer of \(\approx 5 \text{ billion.}\) That’s turns out to be correct, but that number only represents a small fraction of the "sports" industry as compared to things like merchandise and advertising. It can be hard to estimate advertising revenue, but a lot of it comes from sporting goods companies selling things like jerseys or shoes. To get a handle on those things, it’s again best to figure out how much you personally spend on sports (which could mean sporting goods, recreational activities, etc...), and then extrapolate to the rest of the population. Note that if everyone in the US spent just $1.50 a month on something sports-related (a pair of socks from Nike), then that would be 5 billion a year right there.

Let’s once again assume I am typical here, if not in my particular habits, then at least in my total expenditures. I don’t go to very many games, and I don’t spend that much on sporting goods, but I do use the RSF. As a student I don’t have to pay very much for the privilege, but if I were to buy a similar membership on my own it could easily be several hundred dollars/year\(^6\). That has to come from somewhere (tuition), so I should count it under my personal expenditures. Let’s include another few hundred to account for miscellaneous equipment I might buy and say I spend 1000 dollars a year on sports. Like entertainment, that’s also a few percent of my income, which sounds about right.

Not everyone has a gym membership, but other people probably go to games and buy more athletic clothing than I do, so for lack of anything better to do let’s assume everyone else spends money on sports about like I do. If so, sports would be a \(3 \cdot 10^8 \cdot 1000 \approx 300 \text{ billion dollars.}\) industry. An internet search\(^7\) shows the real value to be 441.1 billion.

**Restaurants**

Following the same approach as above, I eat out maybe 4 times a week at an average of 10 bucks per meal. That’s 40 dollars a week, 160 a month, and 1920 dollars per year. If everyone in the country spent that much, the restaurant industry would make \(3 \cdot 10^8 \cdot 1920 \approx 600 \text{ billion dollars.}\) per year. The real revenue for 2008, according to an industry website\(^8\), was 566 billion.

\(^6\)According to their website, a membership at the RSF is 744 dollars a year if you aren’t a student or faculty
\(^8\)http://www.restaurant.org/research/
3 Proposition 13

Proposition 13, written into California's State Constitution in 1978 as Article 13A, limits property taxes to 1% of the assessed value of the property. The assessed value of the property can only change when the property changes ownership. Otherwise, the assessed value remains locked at whatever its value was in 1975. These property taxes are collected by local town and city governments.

Prop 13 has been blamed by some people for a variety of problems in California, perhaps most notably the decline in quality of K–12 education. Before Prop 13, California ranked among the nation's top 5 states for achievement in K–12. Today the state is ranked among the bottom 10 (47th, to be exact).

Put aside the never-ending debate of whether Prop 13 really is to blame for California's decline into ignorance and incompetence, and just ask two questions: How much money derived purely from city property taxes is spent per child for education in Berkeley today? In an alternate universe where Prop 13 did not pass and things stayed as they were in 1978, how much more would it spend today?

How much money does the city collect in property taxes each year? We need to know how many houses are still locked at their property values in 1975, and how many are evaluated at today's (exorbitant) prices.

A look at a recent July/August real estate section of the Sunday San Francisco Chronicle reveals that 20 properties in Berkeley are sold every week. The yearly average must be lower, since nobody likes to show off houses in the dead of winter during the holiday season. We'll take 10 houses per week as a more realistic yearly average. That's 10 \times 50 \sim 500 homes sold per year in Berkeley.

Over the last (2014-1975) = 39 years, about 39 \times 500 \approx 20000 homes have sold. That's roughly 50% of all Berkeley houses, which we estimate to number 4 \times 10^4, roughly 1/(2.5) the population of Berkeley which is 10^5 people (2.5 people per home). A turnover fraction of 50% sounds about right. Of my 4 aunts/uncles who have lived here since 1975, 2 have moved homes. So we'll assume that 50% of homes in Berkeley have exchanged hands since 1975.

How much were property values back then? Today, the same Sunday paper reveals that the median (out of 20 houses) selling price in Berkeley is about $700,000. What were median property values back in 1975? I estimate $70,000. That would correspond to an average appreciation rate of about 6% per year, comparable to the rates that stock market companies like to quote you (5% per year).

So the property tax collected by the city today is 1% \times [(1/2 \times 40000 homes \times 7e4 dollars/home) + (1/2 \times 40000 homes \times 7e5 dollars/home) \sim 1.5 \times 10^8 dollars. Note that we are neglecting the fact that properties are being sold continuously so we really should take care of 1980, 1981, etc. prices. But this is OOM so give us a break.

How much of this goes into education? Well, on tax ballots, doctors + nurses, firefighters + police, and teachers are equally as vocal, so let's say 1/3 goes into education. That gives us \sim 5 \times 10^7 dollars to spend on kids.

How many kids are there? Divide the Berkeley population of \sim 10^5 people into equal time segments, and take K–12 = 13 years out of the human lifespan of 75 years in America. That's 10^5 \times 13/75 \sim 2 \times 10^4 schoolkids. Then the amount of money shelled out per kid is \$2500 just
from city property taxes alone. Web research gives a number of $2000! (again, just from city property taxes alone, which is all that this problem asks for).

(Aside: A look at my property tax bill reveals that since 1978 a few measures (“Measure B”) have passed that serve to boost the amount of money spent on education. For example, there is a fixed tax of about $100 per household towards Berkeley Unified schools. My impression is that these measures are merely band-aids for the problem, and that impression is borne out by my calculation—$100 is a band-aid compared to $2500.)

Alternate Universe: In the universe where Prop 13 did not pass, the revenue sum above would read instead: 3% × [(1 × 40000 homes × 7e5 dollars/home) ∼ 8 × 10^8 dollars, or [6 times more] than in our universe. One factor of 3 comes from the difference between 3% and 1%, and the other factor of 2 comes from the fraction of houses that have not exchanged hands. So instead of $2500, each Berkeley kid would get about $15000 per year. This is a huge improvement (how would you like to be paid 6 times more than what you are currently being paid) — for example, at $15K per year, the Berkeley child would be being supported at practically the same rate that UC Berkeley students are being charged for tuition; they would presumably be getting something closer to a UC Berkeley (i.e., world) class education.

4 The Slow and the Furious

How much rubber is liberated by cars each year driving down Highway 80 from Berkeley to the Maze?

First I will calculate a figure for the amount of rubber liberated per mile driven. You pretty much have to change a car tire every 60,000 miles because of worn treads. The density of rubber is ∼1 g/cm^3. I was taught that you check tread thickness by sticking a penny inside a tread: if you can see Abe Lincoln’s head, then you gotta change your tires. So the tread thickness is a little less than a penny diameter, or about 1 cm. A tire has a radius of ∼12 inches and a width of ∼8 inches. A cylindrical shell approximation reveals that for every 60000 miles driven per tire,

Rubber Lost Per Tire = Volume Lost × Density of Rubber
= 2π × Radius × Width × Tread Depth × Density of Rubber

Plugging in these numbers, ∼4000 grams of rubber are lost for every 60000 miles travelled per 1 tire. This sounds right (gut check). Of course there are 4 tires for every car, so that’s 16 kg of rubber per car per 60000 mi.

From Berkeley (University Avenue exit) to the Maze (80/580/880 interchange) is about 2 miles (if I’m driving 60 mph, I can complete this stretch in 2 minutes.) Bay Area traffic is pretty horrendous — there’s a reason they call this stretch the Maze. So let’s do the rush hour case first. During rush hour, cars are pretty much packed like sardines in a can. Each car is ∼10 feet long (2 prostrate body lengths), so let’s say the intercar distance is ∼15 feet during rush hour. There are 4 lanes on this highway (I actually forget the exact number, but it doesn’t matter to oom), so that means at any given instant, there are 4 × (2 mi/15 ft) ∼ 3000 cars on this stretch. During rush hour it can take an excruciating 30 minutes to travel these 2 miles. Finally rush “hour” takes up about 4 hours out of each day (2 in the morning and 2 in the evening). So during rush hour every day, this 2-mi stretch processes 3000 cars × 4 hr / 30 min ∼ 24000 cars, or equivalently 24000 cars × (16 kg of rubber per car per 60000 mi) × 2 mi ∼ 13 kg per rush-hour-segment-of-1-day.
The other 20 hours of the day are not rush hour. Let’s say 10 of those hours are “full capacity” hours when the highway is at full capacity with cars driving at 60 mph. The intercar spacing is now a lot larger: the DMV 2-second rule means that the intercar spacing is 180 feet. But nobody obeys the 2-second rule, so a more realistic estimate is 90 feet, which we will round to 100 feet. Repeating the exercise above gives $4 \times (2 \text{ mi} / 100 \text{ ft}) \sim 400$ cars at any given instant; a process (flush-out) time of $2 \text{ mi} / 60 \text{ mph} \sim 2$ minutes; a car process rate per full-capacity-segment-of-1-day of $400 \text{ cars} \times 10 \text{ hr} / 2 \text{ min} \sim 120000$ cars; and a rubber-release rate of $120000 \text{ cars} \times (16 \text{ kg of rubber per car per 60000 mi}) \times 2 \text{ mi} \sim 64 \text{ kg per full-capacity-segment-of-1-day}$.

Adding 64 kg to 13 kg = 77 kg. We’ll lazily round up to account for the other remaining 10 night-time hours of a 24-hour day. So that’s 100 kg of rubber per day, or 36500 kg of rubber per year, which we’ll lazily round down to 30000 kg per year or 30 tons per year to account for weekends and holidays when traffic is less. (And that’s just from this little 2-mile segment! Imagine all the rubber released over all the highways in the Bay Area!)

Where does 30 tons of rubber go? It goes onto windowsills; into the ocean; onto the grass; into your lungs.