Problem 1. Protostellar Disk Sizes and the Centrifugal Barrier

The gravitational collapse of a molecular cloud to form a star leads inevitably to the formation of a disk that carries most of the angular momentum of the initial cloud. In this problem we will estimate roughly the sizes of these protostellar disks.

Consider a Jeans-unstable clump of molecular gas of mass $M = 1M_\odot$, radius $R = 0.1$ pc, and spin rate $1$ km s$^{-1}$ pc$^{-1}$. (Clumps like this have been observed via millimeter-wave observations.) As this cloud collapses inward to form a star, the outer radius of the cloud will decrease. The outer radius will not decrease indefinitely, however: It will stall at the centrifugal barrier. Estimate the radius of the centrifugal barrier in AUs, and compare your answer to the sizes of protostellar disks displayed in class.

Problem 2. Making Ice Cubes in the Protoplanetary Disk

(a) Assume that all of the cosmically abundant oxygen in the minimum-mass solar nebula takes the form of water (H$_2$O) vapor. What is the mass density [g/cm$^3$] in water vapor at a stellocentric distance of 5 AU? Use the parameters for the solar nebula described in class.

(b) Estimate the mean thermal speed of water molecules in the solar nebula at this same stellocentric distance. Give your answer in [km/s].

(c) Consider the growth of an ice particle at 5 AU. Assume that the particle accretes water molecules from the gas phase. Estimate the rate at which its radius increases, assuming that water molecules stick to the grain with nearly perfect efficiency due to electrostatic forces and chemical bonds. Express your answer in units of [cm/yr]. Is this calculation promising or discouraging for growing large bodies in the solar nebula, given that the lifetime of the solar nebula is estimated to be $10^7$ yr?

Problem 3. Make like a tree and ...

If all the leaves of a tree fall to the ground, how thick is the layer of leaves on the ground? Give your answer in leaf thicknesses, to order-of-magnitude.

You are lying below the tree. When you look up, the tree presents a certain optical depth to visible light photons coming from the sun. What is the order-of-magnitude
of this optical depth? Why does this answer make sense from the perspective of the vegetation? (Be the tree.)

The above two paragraphs are flip sides of the same question. You can try the second paragraph before the first, or vice versa.

Problem 4. Baby Boom

Landfills are filled with our garbage. In the United States, what fraction of landfills is filled with disposable diapers?