Planetary Astrophysics – Problem Set 1

Due Thursday Sep 10

1 Sea Level

[10 points] The average depth of the Earth's oceans is about $h = 4$ km. How much denser is the water at the bottom of the ocean compared to the top? Neglect variations in salinity and temperature. If this density contrast $\Delta\rho/\rho$ were to (magically) disappear, by about how much would sea level rise? Use a bulk modulus for water of $\mathcal{M}_0 = 0.02$ Mbar.

2 How Round is Round

[10 points] Part of the International Astronomical Union's definition of a planet is that it be massive enough for its own gravity to make it "nearly round". How large must a rock be before self-gravity makes it round? Assume that "nearly round" means a radius deviation $\Delta R/R < f_{\rm crit} = 0.01$.[1] Give a symbolic formula for the minimum "nearly round" radius $R_{\rm min}$ in terms of the failure stress F, the rock density ρ, and $f_{\rm crit}$. Also give a numerical estimate for $R_{\rm min}$ using $F \sim 10^{-4}\mathcal{M}_0$, where the bulk modulus $\mathcal{M}_0 \sim 2$ Mbar, and $\rho \sim 3$ g/cm^3.

3 The Solar Core

(a) [5 points] The core of the Sun is predominantly composed of fully ionized hydrogen having a mass density of $\rho \sim 100$ g/cm^3 and a temperature of $T \sim 10^7$ K. Is the core of the Sun degenerate, or does it behave as an ideal gas? Consider both electrons and protons separately, and justify your answer quantitatively.[2]

(b) [5 points] Show that whether a gas is degenerate or not may be crudely assessed by comparing the thermal de Broglie wavelength λ (the de Broglie wavelength of a particle assuming its momentum is related to its thermal energy) to the mean interparticle spacing x. Particles that are crammed so tightly together that their de Broglie wavelengths strongly overlap are degenerate.

[1] The IAU does not actually give a quantitative criterion for "nearly round".

[2] Please note that the question asks about the Sun. Planets are different! You are encouraged to work out the answer for Jupiter; it might be on an exam.

4 Liquid Giants

[10 points] This problem is adapted from problem 11.2 of Landstreet.

In Saturn, the temperature is about 110 K at a pressure of 0.5 bar (1 bar $= 10^6\,\mathrm{dyne\,cm^{-2}}$*). Below this level the atmosphere is convecting and the temperature increases with depth* z *at the quasi-adiabatic rate of* $dT/dz \approx 0.7\,\mathrm{K\,km^{-1}}$*. Take the atmosphere to be composed purely of molecular hydrogen.*

Gas liquifies once its density approaches that of liquid, of order $\sim 1\,\mathrm{g\,cm^{-3}}$*. At this density, electrons between atoms start to "touch" and become shared in a kind of (electrically conductive) soup. Liquification also requires that the temperature be just right, but here we ignore the temperature dependence and just focus on the density dependence.*

Estimate the depth, z*, at which molecular hydrogen liquifies in Saturn. Use hydrostatic equilibrium, and assume that the gravitational acceleration* g *is constant (it will be, roughly, as long as we don't wander too close to the center of the planet).*

Thereby argue that the gas giant Saturn is more appropriately called a "liquid giant". The same statement holds for Jupiter. Giant planets have oceans of hydrogen.

This problem is an exercise in integrating the equation of hydrostatic equilibrium for an ideal, compressible gas. It does not require one to understand convection or adiabatic lapse rates (we will deal with those concepts later).