Planetary Astrophysics – Problem Set 3

Due Thursday Sep 24

1 M dwarfs vs. G dwarfs

Low-mass M dwarf stars are desirable targets for exoplanet hunting in part because their "habitable zones" (orbital distances where water might be liquid) are smaller than for Sun-like G dwarfs. Smaller habitable zones make for easier hunting grounds for both the Doppler radial velocity technique and the transit technique, as this problem will show.

Define the characteristic orbital radius of the habitable zone (HZ) to be where the effective blackbody temperature of a planet equals a fixed value (say 300 K, but the exact value will not matter for this problem).

Consider a Sun-like star of mass $M = 1M_\odot$ *and a low-mass star of mass* $M' = 0.5M_\odot$*. Assume a stellar luminosity-mass relation* $L \propto M^4$*, and a stellar radius-mass relation* $R \propto M$ *(these relations apply only over a limited range in stellar masses; take a stars course to understand why).*

Calculate [12 points for all questions]:

(a) How much smaller the orbital radius of the M dwarf HZ is compared to the G dwarf HZ. Call this a'/a*. Give a symbolic expression for* a'/a *in terms of* M'/M *and a numerical value for* $M'/M = 0.5$*.*

(b) How much shorter the orbital period is for a planet in the M dwarf HZ vs. the G dwarf HZ. Call this P'/P*; express symbolically in terms of* M'/M *and numerically for* $M'/M = 0.5$*. Assume planet masses are much smaller than stellar masses.*

(c) How much larger the Doppler velocity semi-amplitude is for a planet in the M dwarf HZ vs. the G dwarf HZ. Call this K'/K*; express symbolically in terms of* M'/M *and numerically for* $M'/M = 0.5$*. Assume identical observing conditions between the M dwarf and G dwarf (i.e., identical planet masses, eccentricities, viewing inclinations).*

(d) How much larger the fractional transit depth is for a planet in the M dwarf HZ vs. the G dwarf HZ. Call this $(\Delta F/F)'/(\Delta F/F)$*; express symbolically in terms of* M'/M *and numerically for* $M'/M = 0.5$*. Our notation follows that in lecture;* F *is the stellar flux observed at Earth, and* ΔF *is the amount by which the observed flux decreases because the planet is transiting in front of the star.*

2 Isotropic Orbits

Define the orbital inclination i to be the angle between the orbital plane of an object around a star, and some reference plane (to be specified in instances below).

Consider an isotropic distribution of orbits. By "isotropic" we mean the orbit pole vector (a.k.a. the orbit normal, the vector pointing perpendicular to the orbit plane) can point anywhere with equal probability on the celestial sphere centered on the star.

As one example, long-period comets, with periods longer than about 200 yr, are known to have isotropically distributed orbits (here the reference plane could be any plane, and is often taken to be the "ecliptic", the Earth's orbit plane around the Sun). Long-period comets plunge into the inner solar system from the outer solar system from all directions. Long-period comets originate from the Oort Cloud, a vast reservoir of icy rocks gravitationally perturbed by passing stars and molecular clouds into a spherical shape.

Another example: relative to an observer on Earth, the orbits of extrasolar planets about their host stars should also be distributed isotropically. That is, their orbit normals should be pointing in random directions relative to the observer's line of sight (so in this case the sky plane—the plane perpendicular to the observer's line of sight—is the reference plane). Isotropy is the only reasonable condition because it would be a bizarre cosmic conspiracy to find, say, all exoplanet orbits in the Galaxy to be oriented edge-on relative to an observer on Earth.

(a) [6 points] Derive the differential inclination distribution, dP/di, for isotropic orbits, where dP is the probability of finding an orbit with an inclination between i and $i + di$. Normalize dP/di so that $\int_0^{90^\circ} (dP/di)\, di = 1$ (the probability is 100% that the orbit has an inclination between 0 and 90°).

(b) [3 points] Given your answer in (a), are nearly face-on ($i \approx 0$) orbits more common, less common, or just as common to find as nearly edge-on ($i \approx 90^\circ$) orbits? Justify quantitatively your answer by calculating the relative population of orbits having inclinations between 0–10° and 80–90°.

(c) [3 points] As discussed in class, one way to discover and characterize exoplanets is by measuring the Doppler radial velocity (RV) curve of the host star. The RV curve yields (if the mass of the star is also known) $m \sin i$, where m is the companion mass and i is the a priori unknown inclination of the companion's orbital plane relative to the sky plane. Face-on orbits correspond to $i = 0$, and edge-on orbits to $i = 90^\circ$.

The "planet" HD 217107b is measured to have $m \sin i = 1.3 M_{\rm J}$. Calculate the probability that this planet is not a planet but a brown dwarf whose mass $\geq 13 M_{\rm J}$.

3 Transit Probability

[12 points] A planet orbits a star of radius R_* on a circular orbit of radius a. The planet's radius is very small compared to the star's radius, so we will neglect it; treat the planet as a point particle. Assuming the orbit is oriented randomly (isotropically—see previous problem) relative to an observer on Earth, derive the probability that the

planet is on an orbit that can[1] *transit the star as seen from Earth, using the variables given.*

[1] *Can* transit, not *is* transiting at the time of observation. So do not worry about the duration of the transit.