Planetesimal Formation and Planet Coagulation
Protoplanetary Disks

disk mass $\sim 0.001-0.1$ stellar mass

Wilner et al. 00
Disk surfaces at ~10 AU:
Growth to a few microns

van Boekel et al. 03

Disk interior at ~100 AU:
Growth to a few cm

McCabe, Duchene, & Ghez 03

11.8 μm scattered light

TW Hydra

max s = 1 cm

max s = 1 mm

McCabe, Duchene, & Ghez 03

Calvet et al. 2002
Climbing the size ladder

Grain stopping time $t_{\text{stop}} \equiv \frac{m v_{\text{rel}}}{F_{\text{drag}}}$

Dimensionless stopping time $\tau_s \equiv \frac{\Omega_{\text{Kepler}} t_{\text{stop}}}{\text{Size}}$
Gas-particle entrainment

Assumes minimum-mass solar nebula

\[ \tau_s = \frac{\Omega \cdot t_{\text{stop}}}{10^6} \]

\[ R \text{ [AU]} \]

\[ \lambda_{\text{mfp}} > s \]

\[ v < c_s \]

\[ \rho (c_s - v)^2 \]

\[ \rho (c_s + v)^2 \]

⇒ \[ F_{\text{drag}} \sim \rho c_s v \times \pi s^2 \]

A. Youdin
Grain growth

Sticking $v_{\text{stick}} \sim 1 \text{ m/s for } s \sim 1 \mu\text{m}$

Hertz + surface tension

$$v_{\text{stick}} \sim \frac{4 \gamma^{5/6}}{E^{1/3} \rho^{1/2} s^{5/6}}$$

Terminal $v_{\text{term}} \sim \frac{\rho}{\rho_g} \Omega s$

$\sim 1 \text{ m/s for } s \sim 10 \text{ cm}$

Sticking up to, but not beyond, cm sizes

Chokshi et al. 93, Blum & Wurm 08
Radial drift from headwind

\[ \Omega^2_K r \]

\[ \frac{GM_\ast}{r^2} \]

\[ \Omega^2 r \]

\[ \frac{-1}{\rho_g} \frac{\partial P}{\partial r} \]

\[ \therefore \ \Omega < \Omega_K \]

Meter-sized boulders drift inward from 1 AU within 100 yr

Weidenschilling 77
Nakagawa, Sekiya, & Hayashi 86
Gravitational instability

Disk annulus can start to fragment if
Toomre \( Q \sim 1 \)

\[
\langle \rho \rangle > \rho_{\text{Toomre}} \sim \frac{M_*}{2\pi r^3}
\]

\[ > 10^{-7} \text{ g cm}^{-3} \]

if \( \Sigma_g \sim 2000 \text{ g cm}^{-2} \) (minimum mass solar nebula)

if \( \Sigma_d/\Sigma_g \sim 10^{-2} \) (height - integrated solar metallicity)

then \( \rho \sim \frac{\Sigma_g}{h_g} + \frac{\Sigma_d}{h_d} \)

\[ \sim 3 \times 10^{-9} \text{ g cm}^{-3} \quad \text{if } h_d \sim h_g \]

**Toomre unstable**

\( \rho_d/\rho_g \sim 30 \)

Toomre mass \[ \sim \rho_{\text{Toomre}} \lambda_{\text{unstable}}^3 \]

\[ \sim 10^{19} \text{ g} \, (10 \text{ km}) \]

Clump can resist tidal shear if
Roche unstable

\[
\rho > \rho_{\text{Roche}} \sim \frac{3.5M_*}{r^3}
\]

\[ > 2 \times 10^{-6} \text{ g cm}^{-3} \]

**Roche unstable**

\( \rho_d/\rho_g \sim 600 \)
"Streaming" instability = linear instability between two fluids interacting frictionally in a disk

growth rates peak for $\tau_s \sim 1$

(marginally coupled bodies)

\[
\nabla \cdot \mathbf{v}_g = 0,
\]

\[
\frac{D_p \rho_p}{Dt} = -\rho_p \nabla \cdot \mathbf{v}_p,
\]

\[
\frac{D_p \mathbf{v}_p}{Dt} = -\Omega_K^2 \mathbf{r} - \frac{\mathbf{v}_p - \mathbf{v}_g}{t_s},
\]

\[
\frac{D_g \mathbf{v}_g}{Dt} = -\Omega_K^2 \mathbf{r} + \frac{\rho_p}{\rho_g} \frac{\mathbf{v}_p - \mathbf{v}_g}{t_s} - \frac{\nabla P}{\rho_g}
\]

Youdin & Goodman 05; Johansen & Youdin 07

$\tau_s = 0.1$

$<\rho_d / \rho_g> = 1$
Streaming instability: Physical interpretation

Relies strongly on marginally coupled bodies

Bai & Stone 2011

Jacquet et al. 2011

Roche density
Gravitational instability in the $\tau_s \ll 1$ (small particle) limit

Self-gravity important when

$$\rho > \rho_{\text{Toomre}} \sim \frac{M_*}{2\pi r^3}$$

$$> 10^{-7} \text{ g cm}^{-3}$$

at $r = 1$ AU

if $\Sigma_g \sim 2000 \text{ g cm}^{-2}$ (minimum mass solar nebula)

if $\Sigma_d/\Sigma_g \sim 10^{-2}$ (height integrated solar metallicty)

then $\rho \sim \frac{\Sigma_g}{h_g} + \frac{\Sigma_d}{h_d}$

$$\sim 3 \times 10^{-9} \text{ g cm}^{-3} \quad \text{if } h_d \sim h_g$$

$$\sim 10^{-7} \text{ g cm}^{-3} \quad \text{if } h_d \sim 5 \times 10^{-4} h_g$$

Can the settled “sublayer” achieve Toomre density $< \rho_d/\rho_g > \sim 30$?
Kelvin-Helmholtz instability may limit dust settling

$$\Delta v \sim c_s \frac{c_s}{\nu_K} \sim 25 \text{ m/s nearly independent of } r$$
Necessary criterion for K-H instability in Cartesian shear flow:

Richardson \( Ri \equiv \frac{g \frac{\partial \ln \rho}{\partial z}}{(\frac{\partial v}{\partial z})^2} < Ri_{\text{crit}} = \frac{1}{4} \)

\[ = \frac{\omega_{\text{Brunt}}^2}{(\frac{\partial v}{\partial z})^2} \]

\[ \propto \frac{1}{\Delta z} \left( \frac{1}{\Delta z} \right)^2 \]

\[ \propto \Delta z \]
Numerical simulations of dense midplanes

Initial conditions:
Spatially constant Ri

\[ \mu(z) \equiv \frac{\rho_d(z)}{\rho_g} \]

\[ = \left[ \frac{1}{1/(1 + \mu_0)^2 + (z/z_d)^2} \right]^{1/2} - 1 \]

where \( z_d = \frac{Ri^{1/2} \Delta v}{\Omega} \)

Input parameters:

(Ri, \( \mu_0 \))

(Ri, \( \Sigma_d/\Sigma_g \)) \leftrightarrow (\mu_0, \Sigma_d/\Sigma_g)
Numerical simulations of dense midplanes

EC 08; Lee, EC, Asay-Davis, and Barranco 10a
Toomre-like densities possible for < 4x bulk solar metallicity, or < 4x more mass than minimum-mass solar nebula
Gravitoturbulence? or True Fragmentation?

Constant Ri profiles yield infinite density with self-gravity
Local enrichments of metallicity

Radial pileups

Youdin & EC 04
Toomre density requires:

Super-solar bulk metallicities

\(< 4 \times; \frac{\Sigma_d}{\Sigma_g} = 0.05\)

and/or

Disk masses >
minimum-mass nebula \(< 4 \times\)
Elliptical motion = Guiding center + Epicycle

\( \Omega \) = guiding center (azimuthal) frequency

\( \kappa \) = epicyclic (radial) frequency

“Kepler degeneracy”: \( \kappa = \Omega \)

\( u = \) epicyclic velocity

\( u = \) small body random velocity

Figure 3-6. An elliptical Kepler orbit (dashed curve) is well approximated by the superposition of retrograde motion at angular frequency \( \kappa \) around a small ellipse with axis ratio \( \frac{1}{2} \), and prograde motion of the ellipse’s center at angular frequency \( \Omega \) around a circle (dotted curve).
Massive disks of small bodies
(Goldreich, Lithwick & Sari 04, ARAA)

without gravitational focussing:
\[ t_{\text{acc}} \sim \frac{M}{\dot{M}} \sim \frac{\rho R}{\sigma \Omega} \sim 10^{12} \text{ yr} \]

with gravitational focussing:
\[ t_{\text{acc}} \sim \frac{\rho R}{\sigma \Omega} \left( \frac{u}{v_{\text{esc}}} \right)^2 \sim 10^7 \text{ yr} \]

viscous stirring of small by big
\[ \frac{\rho R}{\Sigma \Omega} \left( \frac{u}{v_{\text{esc}}} \right)^4 \sim \frac{\rho s}{\sigma \Omega} \]

collisional damping of small by small
Extra Slides
Grain growth: Too fast

- No turbulence

Sticks too well

Problem persists even if
- grains are fractal
- monomers are nonspherical

Proposed solution:
Replenishment of micron-sized grains (near-IR opacity) by fragmentation
For small particles well coupled to gas (τs << 1):

1. Is the Richardson criterion a good predictor of stability?
   - Doesn’t formally apply because flow is 3D and rotational
   - Brunt vs. vertical shear vs. Coriolis vs. Kepler shear
   - Coriolis is destabilizing
   - Kepler shear is stabilizing

2. How does maximum dust density ρd depend on bulk (height-integrated) metallicity Σd/Σg?
   - Disk metallicity may be supersolar
   - Host stars of extrasolar planets tend to be metal-rich
   - Planets themselves are metal-rich

Ishitsu & Sekiya 03; Gomez & Ostriker 05; Johnson et al. 10; Guillot et al. 06
Well-coupled gas and dust in a shearing box

\[
\frac{\partial v_x}{\partial t} + \mathbf{v} \cdot \nabla v_x = \frac{-1}{\rho + \rho_d} \frac{\partial P}{\partial x} + 2\Omega_0 v_y + 2q\Omega_0^2 x - \frac{\left(\frac{\partial P}{\partial x}\right)_0}{\rho + \rho_d}
\]

\[
\frac{\partial v_y}{\partial t} + \mathbf{v} \cdot \nabla v_y = \frac{-1}{\rho + \rho_d} \frac{\partial P}{\partial y} - 2\Omega_0 v_x
\]

\[
\frac{\partial v_z}{\partial t} + \mathbf{v} \cdot \nabla v_z = \frac{-1}{\rho + \rho_d} \frac{\partial P}{\partial z} - \Omega_0^2 z
\]

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0
\]

\[
\frac{\partial \rho_d}{\partial t} + \nabla \cdot (\rho_d \mathbf{v}) = 0
\]

\[
\frac{\partial \varepsilon}{\partial t} + \mathbf{v} \cdot \nabla \varepsilon = -(P + \varepsilon) \nabla \cdot \mathbf{v}
\]

\[
P = (\gamma - 1)\varepsilon
\]
Code limitations (so far):

1. No self-gravity
   - Use Toomre density as guide for onset of gravitational instability

2. Dust and gas are perfectly coupled
   - Restricted to studying stability of given initial conditions
Grain growth

\[ mg \sim F_{\text{drag}}(v) \]

\[ \mu s^3 \Omega^2 h \sim \rho_g s^2 c_s v \]

\[ \rightarrow \text{Terminal } v \sim \frac{\mu}{\rho_g} \Omega s \quad \text{(bigger is faster)} \]

Accretion \[ \frac{d}{dt} (\mu s^3) \sim \rho_d v s^2 \rightarrow \dot{s} \sim \frac{\rho_d}{\mu} v \quad \text{(faster is bigger)} \]

\[ \rightarrow \text{Exponential growth } s \sim s_0 \exp(\rho_d \Omega t/\rho_g) \quad \text{(fastest growth in inner disk)} \]

Since \[ t \sim h/v \rightarrow s \sim s_0 \exp(\Sigma_d/\mu s) \]

\[ s_0 \sim 1 \mu m \]
\[ \mu \sim 1 \text{ g cm}^{-3} \]
\[ \Sigma_d \sim 10 \text{ g cm}^{-2} \]

\[ s \sim 1 \text{ cm} \]
\[ t \sim 100 \text{ yr} \]
\[ v \sim 1 \text{ m/s} \]
Relaxing constant $R_i$: Settling from arbitrary initial conditions

1D settling only
$t \sim 10^3 f \left( \frac{0.1 \, cm}{s} \right) \, yr$

Solar Metallicity
$t \sim 10^3 f \left( \frac{0.1 \, cm}{s} \right) \, yr$

1D + 3D

Finding the marginally stable state

Lee et al. 10b
Marginally stable states

Evidence for constant $\langle R_i \rangle$

$\langle R_i \rangle_{\text{crit}} \approx 0.25$

Superlinear relation between midplane density and bulk metallicity

$\langle R_i \rangle_{\text{crit}} \approx 0.5$

$\langle \mu \rangle \propto (\Sigma_d/\Sigma_g)^1$
Grain growth

$\nu_{\text{crit}} \sim 1 \text{ m/s for } s \sim 1 \mu \text{m}$

**Repulsion (elastic modulus $E$)**

Stress $\sigma \sim E \nabla \xi \sim E \frac{\delta}{a}$

$\sigma \sim \frac{mv}{(\delta/v)} \times a^2$

**Repulsive force** $F_R \sim \sigma a^2 \sim \mu^{3/5} E^{2/5} s^2 v^{6/5}$

**Repulsive energy** $U_R \sim F_R \delta$

**Adhesion (surface tension $\gamma$)**

Binding energy $U_B \sim \gamma a^2$

$U_R = U_B \rightarrow \nu_{\text{crit}} \sim 4 \frac{\gamma^{5/6}}{E^{1/3} \mu^{1/2} s^{5/6}}$

Hertz 1882, Chokshi et al. 93
Rotational Effects

Coriolis

Kepler radial shear

\[ 2\Omega \]

destabilizing
Cabot 84
Gomez & Ostriker 05

\[ \frac{3}{2}\Omega \]

stabilizing
Ishitsu & Sekiya 03
EC 08
Barranco 09

Brunt oscillation

\[ < \Omega \]

stabilizing

Vertical shear

\[ < \Omega \]

destabilizing

Ri