Goals

- Refresh your memory about statistical mechanics
- Practice scaling arguments

Statistical Mechanics

1. Suppose we have a non-relativistic, classical, ideal gas with a constant temperature $T$. These particles obey Maxwell-Boltzmann statistics.

   a) Roughly, what do each of the qualifiers “non-relativistic”, “classical”, and “ideal” mean?

   b) How does the probability of a particle being in a particular state with energy $E$ scale with $E$ and/or $T$?

   c) What is the energy of a particle in a non-relativistic, classical, ideal gas?

   d) Suppose we have a particle with energy $E$. Sketch where it would be located in momentum space.

   e) What if we have a particle with energy between $E$ and $E + \Delta E$? What is the volume of the region in momentum space where it could be?
(f) Previously, you wrote down the probability of a particle being in a particular state $E$. Assume that the number of states with energy between $E$ and $E + \Delta E$ is proportional to the volume in momentum space you just found. What is the probability of a particle being in any state with energy between $E$ and $E + \Delta E$?

Classically, perhaps it’s not obvious that the ‘size’ of a state is some constant amount of momentum space. Quantum mechanically, you can argue this from the uncertainty principle.

(g) Your previous answer should have been in the form $f(E)\Delta E$. This is a **probability distribution function**, which represents the probability of finding a particle with and energy between $E$ and $E + \Delta E$. To make this an equality rather than a proportionality, we would need to integrate this from $E = 0$ to $E = \infty$ and normalize such that it is equal to one.

$$\int_{0}^{\infty} f(E)dE = 1.$$ 

Don’t do the integral, but keep in mind that a probability distribution integrated over all possibilities must always equal one.

The answer you would find is

Once you’re here, let me know, and I’ll tell you the normalized function.

Sketch this function and make a guess where the peak is.

(h) Suppose we wanted to do something similar for photons. Which of our initial assumptions aren’t valid anymore? What sorts of things would we have to do differently?
Scaling Arguments

2. In lecture on Tuesday, we wrote down the equation of hydrostatic equilibrium

\[ \frac{dP}{dr} = -g \rho \]

and applied it to an isothermal atmosphere.

(a) What do we mean when we say ‘scale height’?

Explain qualitatively; don’t write down the expression we derived in class.

(b) Use your definition of ‘scale height’ to make an approximation for the value of \( \frac{dP}{dr} \).

You’re going to want to get comfortable with approximations like these because we’re going to be using them a lot.

(c) Use your approximation to solve an algebraic expression for the scale height, assuming that the pressure is gas pressure.

(d) Write your expression for the scale height as a scaling relation, normalized to values appropriate for the earth’s atmosphere.

Do things at the order of magnitude level. Now is also a good time to practice using cgs units.

(e) What if the pressure in the earth’s atmosphere were radiation pressure instead of gas pressure?