Post-Main-Sequence Evolution

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Goals

• Review the post-main-sequence evolution of a $1\,M_\odot$ star
• Review the Chandrasekhar mass

Post-Main-Sequence Evolution

1. We’re going to do the same HR diagram exercise as before, except now we’re going to include the phases of stellar evolution after the main sequence.

(a) Draw a large HR diagram on the board. Draw the Main Sequence. Draw the Hayashi line.
(b) Draw the path of star formation for a $1M_\odot$ star.
(c) What happens to this star while on the Main Sequence?
(d) Draw the path of the post-main-sequence evolution.
(e) Label the Red Giant Branch, Asymptotic Giant Branch, and Planetary Nebula phases.
(f) Label where shell fusion occurs. Is it H fusion or He fusion?
(g) Label where helium (core) fusion occurs.
(h) Label where significant mass loss occurs.
(i) Label where degeneracy pressure dominates.
(j) Label the evolutionary path of a white dwarf. Why does it behave this way?
On the RGB, the shell fusion is just H. On the AGB, it is H and He.
Chandrasekhar Limit

2. From our discussions in lecture, we know that you reach the Chandrasekhar limit when the star becomes supported by relativistic degeneracy pressure and the total energy becomes zero. For a star composed of equal numbers of protons and electrons:

(a) Estimate the Fermi energy.

**Solution:** Each quantum state is filled with a fermion. The approximate size of a state in phase space is $\sim h^3$. If we have $N_f$ fermions, then the amount of phase space occupied will be $\sim N_f h^3$. The volume of phase space is just $R^3 p_f^3$, because the fermions are contained in a sphere of radius $R$ in physical space and a sphere of radius the Fermi momentum in momentum space. Since we’re relativistic, $E_f = p_f c$. Rearranging,

$$E_f \sim \frac{\hbar c}{R} N_f^{1/3}$$

(b) Estimate the gravitational binding energy (per particle).

**Solution:** The total mass can be written as $N_m m$, where $m$ is the mass of the particle which is dominating the mass of the system and $N_m$ is the number of those particles. (For a WD, this would be the proton.) So the binding energy for one particle is

$$E_b \sim \frac{G (N_m m) m}{R}$$

(c) By setting these equal, you should find a critical number of particles. Use this to estimate a mass.

**Solution:** Setting the binding energy equal to the equal to the Fermi energy is finding when the total energy of the system is zero. (Previously, we’ve shown that this is going to be unstable.) For a white dwarf, $N_m = N_f$ from charge neutrality, so we’ll just call this $N$.

$$\frac{\hbar c}{R} N^{1/3} \sim \frac{G N m^2}{R}$$

and solving for $N m$

$$N \sim \left( \frac{\hbar c / G}{m_p} \right)^{3/2} \sim 10^{57}$$
The total mass will be

\[ M \sim m_p N \sim 10^{33} \text{ g} \sim 1 \, M_\odot \]

(d) What would be different if the star were entirely neutrons?

**Solution:** If the star were entirely neutrons, the neutrons would provide both the mass (so \( N_m = N_n \)) and the degeneracy pressure (so \( N_f = N_n \)). This means \( N_m = N_f \) again, but for a different reason. Also, \( m = m_n \), but since \( m_n \approx m_m \), nothing about our quantitative estimates will change.