Nuclear Reactions & Scaling Arguments

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Goals
• Review nuclear reaction rates
• Practice using scaling arguments

Nuclear Reactions

1. Consider the simple reaction $A \xrightarrow{k_1} B \xrightarrow{k_2} C$. The $k_i$ are the rate constants for each step. A larger $k$ means a faster reaction.

   (a) Describe what happens when $k_1 \gg k_2$. What is the effective rate constant for the conversion of $A$ to $C$?

   (b) Describe what happens when $k_1 \ll k_2$. What is the effective rate constant for the conversion of $A$ to $C$?

2. Why do reaction rates depend on density in the way that they do?

   (a) Explain in words and/or pictures why the reaction rate for something like $p + p \rightarrow D$ scales like $n_p^2$.  

   Think in microscopic terms.
(b) Suppose that the reaction $4 \times ^1\text{H} \rightarrow ^4\text{He}$ happened by the simultaneous collision of 4 protons. How would $\varepsilon$ scale with $\rho$?

3. Frequently, we approximate nuclear reaction rates with the simple form $\varepsilon \propto \rho^\alpha T^\beta$. On this plot of $\varepsilon$ vs. $T$ (at fixed $\rho$), do the following:

(a) Mark the central temperature of the sun
(b) Explain the meaning of $\beta$ by sketching on the graph
(c) Find an approximate value of $\beta$ (at $T_{\odot}$)
Scaling Arguments

4. In lecture, Eliot mentioned that fusion in the solar corona is negligible. We want to confirm his statement and practice exploiting the convenience of scaling arguments. This plot gives us a rough structure of the corona.

![Diagram showing the temperature and number density profile of the solar corona](https://www.cfa.harvard.edu/~scranmer/Ay201a/handout_dougherty.pdf)

(a) What is the amount of mass in the solar corona? Approximate it as a spherical shell of constant density.

You should get a rough density, radius and thickness from the plot.

(b) At fixed temperature, write down a proportionality for the hydrogen fusion luminosity in terms of ρ and M.

Don’t try and relate ρ to M. In doing so, what would you be assuming?

(c) Estimate the luminosity (from fusion) of the solar corona.

Use what you know about the sun to anchor a scaling relation.