Ideal Electron Gas & Absorption Lines

2 November 2011

Goals

- Review some different regimes of an ideal electron gas
- Explain stellar absorption lines

EoS of Electron Gas

1. We’ve been discussing the equation of state (EoS) of an ideal electron gas. The necessity of considering relativity and quantum mechanics means that at a first pass, we can divide $(\rho, T)$ space up into four regions, where different physics is important.

- Classical, non-relativistic
- Classical, ultra-relativistic
- Degenerate, non-relativistic
- Degenerate, ultra-relativistic

Assuming that $\mu_e = 1$, then these four regimes are

![Graph showing the four regimes of electron gas EoS](image-url)
(a) What is $P(\rho, T)$ in each of these regimes?

(b) Quantitatively, what are the 4 lines drawn on the plot? Ignore factors of order unity, but write each relation in terms of fundamental constants.
Absorption Lines

2. Complete the following table by indicating how the values of the physical quantities at the line center (subscript 0) compare to the nearby continuum values (no subscript).

<table>
<thead>
<tr>
<th></th>
<th>At line</th>
<th>Off line</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu = \nu_0$</td>
<td>$\sigma_0$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>$\ell_0$</td>
<td>$\ell_0$</td>
<td>$\ell$</td>
</tr>
<tr>
<td>$R_{ph,0}$</td>
<td>$R_{ph,0}$</td>
<td>$R_{ph}$</td>
</tr>
<tr>
<td>$T_{eff,0}$</td>
<td>$T_{eff,0}$</td>
<td>$T_{eff}$</td>
</tr>
<tr>
<td>$I_{\nu_0}$</td>
<td>$I_{\nu_0}$</td>
<td>$I_{\nu}$</td>
</tr>
</tbody>
</table>

$I$ is the brightness at that frequency.

3. Let’s estimate the lowest temperature where we could see Balmer absorption lines. Remember that the line cross-section is approximately $10^8$ times greater than the continuum cross section.

(a) Write a relationship between the mean free paths at & off the line which means we will (just) no longer see the line.

(b) At a given temperature, what fraction of atoms are in the $n = 2$ state? Assume that the temperature is low enough that almost everything is neutral and in the $n = 1$ state.

(c) Combining (a) & (b), what’s the lowest temperature at which we could see Balmer lines?