Control Interface for Delay, Amplitude, and Phase of IF Processor

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This memo summarizes the results of a meeting which took place on 7/16/3 attended by Dave DeBoer, G. Girmay-Kaleta, Rob Ackermann, and Gerry Harp. The goal of this meeting was to choose the layout of the communication network between the ATA “host” computers and the IFProcessors. It begins with some ancillary calculations which justify the figures arrived at.

LEO Angular Velocity and Derivatives

It is now common knowledge that the quantity of information that needs to pass from the host to the IFP is dominated by setting the delay (τ) and complex gain (represented as amplitude and phase angle (α, ϕ), respectively). It is agreed that at the host/IFP interface, we shall specify these three quantities with a Taylor’s series expansion nominally with two terms each (i.e. τ,  τ, α,  α, ϕ,  ϕ), although we suggest that three terms should be considered for (ϕ).

The upper limit for tracking speed of the dishes or the IFP is set by the requirement to track LEO satellites. A memo on the ATA software website, “Az/El Trajectories and Spline Interpolation, (section: Useful Parameters)” outlines tracking calculations for LEO’s and we reference it here. The angular velocity  , angular acceleration  , and angular jerk  of a LEO passing through zenith are approximated in terms of elevation angle θ by

\[ \dot{\theta} = \frac{v}{h} \sin^2 \theta, \quad [1] \]

\[ \ddot{\theta} = \frac{2v^2}{h^2} \sin^3 \theta \cos \theta, \quad [2] \]

and

\[ \dddot{\theta} = \frac{2v^3}{h^3} (3 - 4 \sin^2 \theta) \sin^4 \theta. \quad [3] \]

Specifying Delay

For an antenna at the extreme edge of the array (d = 500 m from center), the signal path length difference relative to the array center is  p = d \cos \theta , leading to a signal delay of
\[ \tau = \frac{d}{c} \cos \theta = 1.6 \times 10^{-6} \cos \theta \text{ seconds.} \quad [4] \]

It is possible, but not very illuminating to work out all the derivatives of \( \tau \). Instead we’ll work with easily calculated upper bounds (actual values are within a factor of 2). The LEO velocity is practically independent of its height, \( v \approx \sqrt{rg} = 7900 \text{ m/s} \), and we retain consistency with the previous memo by using a LEO height \( h = 350 \text{ km} \). Also notice we assume an IF sampling rate of 150 MHz.

\[ \tau_{\text{max}} \sim \frac{d}{c} = 2 \times 10^{-6} = 300 \text{ samples}, \]
\[ \dot{\tau}_{\text{max}} \sim \frac{dv}{ch} = 4 \times 10^{-8} = 6 \text{ samples/s}, \]
\[ \ddot{\tau}_{\text{max}} \sim \frac{2dv^2}{ch^2} = 2 \times 10^{-9} = 0.3 \text{ samples/s}^2, \text{ and} \]
\[ \dddot{\tau}_{\text{max}} \sim \frac{6dv^3}{ch^3} = 1 \times 10^{-10} = 0.015 \text{ samples/s}^3. \quad [5] \]

We wish to set the delay with sub-sample accuracy, say 1 nanosecond (this is 1/6 the sampling period). To justify this choice, we must consider the phase error this introduces for a signal at the edge of the IF band, that is, 50 MHz away from the tuning center*. Such a signal will have a maximal phase error \( \Delta \varphi_{\text{max}} = 3^\circ \). The detected power is proportional to \( \cos^2(\Delta \varphi_{\text{max}}) \), or 0.996. However, for any integration period longer than a millisecond, it is appropriate to calculate the time average power for all possible values of phase error, so the relevant number is \( < \cos^2(\Delta \varphi) > = 0.999 \), which differs from unity by a negligible amount.

With 1 nanosecond accuracy, the total delay range can be represented by 4000 different values including both positive and negative delay. For convenience, we propose to represent delay with a signed 16-bit value indicating nanoseconds, giving maximal delays of +/- 30 \( \mu \text{s} \) or supporting an array of radius 9 km.

If we chose to specify \( \tau \) directly every time it was necessary, then from eq. [5] it must be specified about 40 times per second:

\[ \text{max error in } \tau = 1 \text{ ns} = \dot{\tau} t_{\text{max}} \Rightarrow t_{\text{max}} = 0.025 \text{ s}, \]

* The argument presented here is due to Larry D’Addario.
where \( t_{\text{max}} \) is the maximal amount of time we can use a given setting. Instead, we propose to specify \( \tau(t) \) in a two-term Taylor’s series:

\[
\tau(t) = \tau_0 + \dot{\tau} t.
\]  \[6\]

We propose to specify \( \dot{\tau} \) as a signed 16-bit value indicating \( 10^{-11} \) seconds / sec. With this choice, new values of \( \tau_0 \) and \( \tau \) need be specified only once per second as seen from:

\[
\text{max error in } \tau = 1 \text{ ns} = \frac{1}{2} \dot{\tau} t_{\text{max}}^2 \Rightarrow t_{\text{max}} = 1 \text{ s}.
\]

Notice also that with this choice, the maximal \( \tau \) that can be specified is comfortably 10 times a large as required, and a given value of delay rate could be used for up to 100 seconds before rounding errors begin to affect system response. Another point to consider is that in sidereal tracking, the maximal delay rate is \( 10^3 \) times smaller than that for a LEO, leading to a small but nonzero value for \( \dot{\tau} \).

**Specifying Phase**

After the delay, each signal path is multiplied by a single complex coefficient, \( c \), which we represent as an amplitude \( \alpha \) and phase \( \phi \):

\[
c = \alpha \exp(-i \phi).\]

\[7\]

We begin with an examination of the phase parameter \( \phi \). We use \( \phi \) to steer a phased array beam to any position on the sky independent of the delay setting. This ability is required because multiple beams will share a single delay. Hence we can pursue a derivation parallel to the previous section, assuming that we don’t have any delay at all and we must line up our signals using phase only.

For a given frequency \( f \), the relationship between delay and phase is linear

\[
\phi(t) = 2\pi f \tau(t).
\]  \[8\]

Using \( f_{\text{max}} = 11.2 \text{ GHz} \), we can immediately conclude that

\[\text{The requirements for phase accuracy are in some ways more stringent than for delay accuracy. The delay accuracy is calculated using a frequency related to the total bandwidth of the observation (in this case, 50 MHz at the band edge). The phase, on the other hand, is applied at the sky frequency (11.2 GHz), so a much greater dynamic range is required.}\]
\[ \phi_{\text{max}} \sim 2\pi f_{\text{max}} \frac{d}{c} = 1 \times 10^5 \text{rad} = 7 \times 10^6 \text{deg}, \]
\[ \dot{\phi}_{\text{max}} \sim 2\pi f_{\text{max}} \frac{dv}{ch} = 3 \times 10^4 \text{rad/s} = 2 \times 10^5 \text{deg/s}, \]
\[ \ddot{\phi}_{\text{max}} \sim 4\pi f_{\text{max}} \frac{dv^2}{ch^2} = 100 \text{rad/s}^2 = 7 \times 10^3 \text{deg/s}^2, \text{and} \]
\[ \dddot{\phi}_{\text{max}} \sim 12\pi f_{\text{max}} \frac{dv^3}{ch^3} = 10 \text{rad/s}^3 = 5 \times 10^2 \text{deg/s}^3. \]

Of course, the phase is specified modulo $2\pi$.

Next we must specify the accuracy for the phase. Similar to the delay calculation, a phase accuracy on the order of $1^\circ$ is adequate. Studies of beamforming (see memo: “Customized Beamforming at the ATA”) indicate that phase errors on this level are easily tolerated, and what is good for beams is good for the correlator (based on principle that a beam is a 1 pixel correlator).

Using this value, here are the maximum amounts of time that a single Taylor’s series expansion of phase can be used for varying numbers of terms:

1 Term : \[ t_{\text{max}} = \frac{\Delta \phi}{\dot{\phi}} = 7 \times 10^{-6} \text{ s} \]

2 Terms : \[ t_{\text{max}} = \sqrt{\frac{2\Delta \phi}{\ddot{\phi}}} = 2 \times 10^{-2} \text{ s} \] \[ \text{[10]} \]

3 Terms : \[ t_{\text{max}} = \sqrt[3]{\frac{6\Delta \phi}{\dddot{\phi}}} = 0.2 \text{ s} \]

With a two-term Taylor’s series it is necessary to recalculate the series expansion at 100 Hz, but if we specify a three-term Taylor’s series, then the calculation need be performed only at 10 Hz rate. The latter case has a 50% increase in data content but is sent only 1/10 as often, resulting in a net reduction in aggregate data rate by a factor of 6. We therefore suggest that this approach is adopted.

For the three-term case, the Taylor’s series expansion is

\[ \phi(t) = \phi_0 + \dot{\phi} t + \frac{1}{2} \ddot{\phi} t^2. \] \[ \text{[11]} \]

We propose that $\phi_0$ is specified with a signed 32-bit value representing units of $0.001^\circ$. Admittedly, this is overkill and a 16-bit value representing $0.1^\circ$ would serve as well. However, we shall see that $\dot{\phi}$, for $\dddot{\phi}$ are best represented by 32-
bit values, so for the sake of consistency and beauty, we prefer this choice. We prefer degrees to radian coordinates since a single revolution can be expressed exactly (360000 units), hence modulo operations are simplified.

For $\phi$, we propose a signed 32-bit value representing units of $0.001^\circ / s$. The maximum phase rate that can be expressed with such a value is $3 \times 10^5$ rad / s, or 100x larger than required by eq. 9. It is not impossible to use a 16-bit value here, but such representations would “expire” quickly resulting in a higher aggregate data rate, and there would be no slack in the dynamic range. Arguably, a 24-bit number could be used, but we prefer to use “standard” representations (8, 16, 32, 64) unless circumstances demand otherwise. For sidereal tracking, the phase rate is once again reduced by at least $10^3$, and is on the order of $50^\circ / \text{sec}$.

For $\phi$ a 16-bit value is barely sufficient, but provides no slack in dynamic range. A more comfortable choice is therefore a signed 32-bit value representing units of $0.001^\circ / s^2$.

If there is a compelling reason to reduce the data rate at the end, a workable but slightly less convenient representation of phase values would be signed 24-bit values indicating units of $0.1^\circ, 0.1^\circ / s, 0.1^\circ / s^2$.

**Specifying Amplitude**

For ordinary tracking of any single point on the sky, $\alpha$ is always unity. However, certain astronomical observations demand the ability to “taper” the phased array beam, which implies control of $\alpha$. Likewise, certain RFI mitigation techniques that we hope to test require control of $\alpha$.

One can view $\alpha$ as representing a kind of phase by equating

$$c = \alpha \exp(-i \phi) \equiv \exp(\varphi_a - i \varphi).$$

[12]

Just as with $\phi$, we require control of $\varphi_a$ to an accuracy of about $1^\circ$. We therefore propose to specify $\alpha$ with a two-term Taylor's series expansion

$$\alpha(t) = \alpha_0 + \dot{\alpha}t$$

[13]

where both $\alpha_0$ and $\dot{\alpha}$ are represented as signed 16-bit numbers in units of $0.001$. With this choice, unit is represented as the number 1000.

To further justify this choice, consider the following application. One could calibrate a beamformer on a strong point source by giving 349 antennas $\alpha = 0.1$ and giving the last antenna $\alpha = -34.9$. With this choice, the phase of the last antenna can be varied so as to minimize the beamformer output, resulting in the
optimal phase for that antenna. This process could then be repeated with each antenna in turn and perhaps iterated, finally converging on a calibration of all antenna phases. This process could not be performed if we had chosen an 8-bit value for $\alpha$.

**Aggregate Data Rate**

With the above mentioned choices, it is necessary to recalculate and transmit parameters no more than 10 times per second. We shall tag each of these updates with a 16-bit “sequence” number to allow the IFP to verify that it is getting every sample. To completely specify delay, phase, and amplitude in one calculation we need

$$(\text{delay} = 32 \text{ bits}) + (\text{phase} = 96 \text{ bits}) + (\text{amplitude} = 32 \text{ bits}) + (16\text{-bit sequence #}) = 176 \text{ bits per update}.$$

resulting in 1760 bits per second. For each of 350 antennas, updates must be applied to 32 single-polarization beams, for an aggregate data rate of 19.7 Mb/s.

**Packetization**

The destination for this data is the IFP boards, each of which processes 16 single-pol beams. There are 12 IFP boards per crate, 8 crates per rack, and 8 racks, which multiplies to a total of 12288 beams total which is approximately equal to 350 * 32 beams. This layout, if fully populated, could support up to 384 antennas.

We choose to packetize the data in the following way. Packets are addressed to a single crate, which represents 192 beams. Each packet contains approximately 34 kbits, and is issued 10 times per second via UDP. Each UDP message is sent upon receipt of a hardware trigger that is generated from the 1 PPS signal distributed throughout the system.

With this approach, each IFP receives a packet that is addressed to the entire crate. It can use either the 1 PPS or a multiplied version of this signal to identify when each packet should be applied. The IFP will ignore data not directly addressed to itself.

The figure below describes the system layout and data rates at each level. It is drawn with a 1 to 1 correspondence between packetizers and racks, although this ratio could change without affecting the layout.
Host Generates Abstract Trajectories

10 Hz Trajectories in UDP Packets

-20 Mb/s

1 GB Ethernet

Rack Switch

2.4 Mb/s

340 kB/s

340 kb/s

8 Packetizers and Racks

8 Crates per Rack

12 IFP's per Crate

(No more expansion here)

X10

1 pps

X10

1 pps
## Summary

<table>
<thead>
<tr>
<th><strong>QUANTITY</strong></th>
<th><strong>VALUE</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{\text{LEO}}$</td>
<td>$\sim 7900 \text{ m/s}$</td>
</tr>
<tr>
<td>$h_{\text{LEO}}$ (lowest LEO)</td>
<td>350 km</td>
</tr>
<tr>
<td>$\dot{\theta}_{\text{max}}$</td>
<td>$0.022 \text{ rad/s} = 1.3^\circ /\text{s}$</td>
</tr>
<tr>
<td>$\ddot{\theta}_{\text{max}}$</td>
<td>$0.0010 \text{ rad/s}^2$</td>
</tr>
<tr>
<td>$\dot{\theta}_{\text{sideral, max}}$</td>
<td>$7.27 \times 10^{-5} \text{ rad/s} = 0.00417^\circ /\text{s}$</td>
</tr>
<tr>
<td>$\tau_{\text{max}}$ = Max delay (500 m)</td>
<td>$2 \times 10^6 \text{ s}$</td>
</tr>
<tr>
<td>$\dddot{\tau}_{\text{max}}$</td>
<td>$4 \times 10^{-8} \text{ s} / \text{s} = 6 \text{ samples / s}$</td>
</tr>
<tr>
<td>$\dddot{\tau}_{\text{max}}$</td>
<td>$2 \times 10^{-9} \text{ s} / \text{s}^2 = 0.3 \text{ samples / s}^2$</td>
</tr>
<tr>
<td>$\dddot{\tau}_{\text{max}}$</td>
<td>$1 \times 10^{-10} \text{ s} / \text{s}^3 = 0.015 \text{ samples / s}^3$</td>
</tr>
<tr>
<td>$\phi_{0}$ representation</td>
<td>16-bit signed integer (nanoseconds)</td>
</tr>
<tr>
<td>$\dot{\phi}$ representation</td>
<td>16-bit signed integer (10$^{-11}$ seconds / s)</td>
</tr>
<tr>
<td>$\phi_{\text{max}}$ = Max phase across array</td>
<td>$7 \times 10^6 \text{ degrees}$</td>
</tr>
<tr>
<td>$\dot{\phi}_{\text{max}}$</td>
<td>$2 \times 10^5 \text{ deg / s}$</td>
</tr>
<tr>
<td>$\ddot{\phi}_{\text{max}}$</td>
<td>$7 \times 10^3 \text{ deg / s}^2$</td>
</tr>
<tr>
<td>$\dddot{\phi}_{\text{max}}$</td>
<td>$5 \times 10^2 \text{ deg / s}^3$</td>
</tr>
<tr>
<td>$\phi_{0}$ representation</td>
<td>32-bit signed integer (millideg)</td>
</tr>
<tr>
<td>$\dot{\phi}$ representation</td>
<td>32-bit signed integer (millideg / s)</td>
</tr>
<tr>
<td>$\ddot{\phi}$ representation</td>
<td>32-bit signed integer (millideg / s$^2$)</td>
</tr>
<tr>
<td>$\omega_{0}$ representation</td>
<td>16-bit signed integer (0.001)</td>
</tr>
<tr>
<td>$\dot{\omega}$ representation</td>
<td>16-bit signed integer (0.001 / s)</td>
</tr>
<tr>
<td>Aggregate Data Rate</td>
<td>20 Mbit / s</td>
</tr>
<tr>
<td>Data rate to rack</td>
<td>2.4 Mbit / s</td>
</tr>
<tr>
<td>Data rate to crate</td>
<td>340 kbit / s</td>
</tr>
<tr>
<td>Data rate to IFP board</td>
<td>340 kbit / s</td>
</tr>
<tr>
<td>Single Pol Beams per Antenna</td>
<td>32</td>
</tr>
<tr>
<td>Signal paths per IFP board = 4 Single Pol Beams x 4 Antennas</td>
<td>16</td>
</tr>
<tr>
<td>IFP boards per crate</td>
<td>12</td>
</tr>
<tr>
<td>Crates per rack</td>
<td>8</td>
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<tr>
<td>Racks for 384 antennas</td>
<td>8</td>
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</table>