Lecture 25
Physical Properties of Molecular Clouds

1. Giant Molecular Clouds
2. Nearby Clouds
3. Empirical Correlations
4. The Astrophysics of the X-Factor

References
Blitz & Williams, “Molecular Clouds
Myers, “Physical Conditions in Molecular Clouds”
in Origins of Stars & Planetary Systems eds. Lada & Kylafis
http://www.cfa.harvard.edu/events/1999/crete
Bergin & Tafalla, ARAA 45 339 2007 - observations of cold
dark clouds
McKee & Ostriker, ARAA 45 565 2007 - summary of
observations and theoretical interpretations

1. Giant Molecular Clouds

• An important motivation for studying molecular
  clouds is that’s where stars form
• Understanding star formation starts with
  understanding molecular clouds
• In addition to their molecular character, large
  and massive molecular clouds are dynamical
  systems that are
    Self-Gravitating
    Magnetized
    Turbulent
• The central role of gravity high distinguishes
  them from other phases of the ISM.
What is a Molecular Cloud?

- Molecular clouds have dense regions where the gas is primarily molecular.
- Giant molecular clouds (GMCs) are large clouds with $10^4 M_\odot < M < 6 \times 10^6 M_\odot$ sizes in the range 10-100 pc.
- The filing factor of GMCs is low; there about 4000 in the Milky Way. They have as much atomic as molecular gas.
- Mean densities are only $\sim 100$ cm$^{-3}$, but molecular clouds are inhomogeneous and have much higher-density regions called clumps and cores.

NB There is no accepted explanation for the sharp upper limit to the mass of GMCs; tidal disruption and the action of massive stars have been suggested.

2. The Orion Molecular Cloud Complex

These clouds can’t be much older than 10-20 Myr, the age of the oldest OB sub-association.

Associations older than 20-30 Myr are not associated with GMCs.
Orion: The Very Large Scale Picture

Dame et al. (2001)  
CO survey

See the next slides with stars.
Large-scale Optical and CO Images

Orion Molecular Clouds A and B in CO
Constellation Scale Optical and CO Images
Orion Molecular Clouds A and B in IR Constellation Scale Optical and IRAS Images

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Summary for Orion GMCs

- Cloud A (L1641) exhibits typical features of GMCs:
  - fairly well defined boundaries: GMCs seem to be discrete systems
  - clumpy, but with unit surface filling factor in optically thick $^{12}\text{CO}$ 1-0 in low resolution maps
  - elongated, parallel to the plane of the Galaxy
  - strong velocity gradient (rotation)
- Star clusters form in GMCs
  - no local GMCs ($d < 1\ \text{kpc}$) without star formation
  - one nearby GMC ($d < 3\ \text{kpc}$) without star formation (Maddalena’s cloud $\sim 10^5\ M_\odot$)

Essentially all star formation occurs in molecular clouds

3. Basic Properties of Molecular Clouds

- Important deductions can be made from CO studies of molecular clouds by very direct and simple means.
- The relevant data are the line width, the integrated line strength and the linear size of the cloud.

For a Gaussian line, the variance or dispersion $\sigma$ is related to the Doppler parameter $b$ and the FWHM as follows:

$$\sigma = b/2^{1/2}, \quad \text{FWHM} = 2 \sqrt{(2 \ln 2) \sigma} \approx 2.355 \sigma$$

For thermal broadening,

$$b_{\text{th}} \approx 0.129 \left(\frac{T}{A}\right)^{1/2} \text{km s}^{-1} \quad (A = \text{atomic mass})$$

More generally, in the presence of turbulence,

$$\sigma^2 = \frac{kT}{m} + \sigma_{\text{turb}}^2$$
Application of the Virial Theorem

A key step in the elementary interpretation of the CO observations by Solomon, Scoville, Sanders et al. uses the virial theorem, which assumes that

**GMCs are gravitationally bound in virial equilibrium,**

The virial theorem with only gravitational forces reads:

\[
-\langle V \rangle = 2 \langle K \rangle = \langle mv^2 \rangle \quad \text{or} \quad \langle \frac{GM}{R} \rangle = \langle v^2 \rangle = \sigma^2
\]

Measurements of the size \( R \) and the velocity dispersion \( \sigma \) can then be used to estimate the mass of the GMC:

\[
M \approx \frac{R \sigma^2}{G}
\]

The Linewidth-Size Correlation

- \( T_{\text{kin}} \approx 20 \text{ K} \Rightarrow \sigma < 0.1 \text{ km/s} \) (from low-\( J \) CO lines)
- Linewidths are suprathermal
- Noticed by Larson (MNRAS 194 809 1981), who fitted \( \sigma \sim S^{0.38} \) close to Kolmogorov 1/3.
- Others found \( \sigma \sim S^{0.5} \) (\( \sigma \) in km s\(^{-1}\) and \( S \) in pc).
- The correlation extends to smaller clouds and smaller length scales within GMCs (Heyer & Brunt, ApJ 615 L15 2004), but not to cores
- If the linewidth is a signature for turbulence*, this correlation is an **empirical statement about turbulence in molecular clouds.**

* For an introduction to interstellar turbulence, see Sec 2. McKee & Ostriker (2007)
The Luminosity-Mass Correlation

$$I_{CO} = \int_{\text{line}} T_A(v) dv$$

is the line integrated intensity for optically thick $^{12}$CO. The CO luminosity of a cloud at distance $d$ is

$$L_{CO} = d^2 \int_{\text{cloud}} I_{CO} d\Omega; \quad \text{hence} \quad L_{CO} \approx T_{CO} \Delta v \pi R^2$$

where $T_{CO}$ is the peak brightness temperature, $\Delta v$ is the velocity line width and $R$ is the cloud radius.

Substituting $\Delta v^2 \approx \frac{GM}{R}$ (virial equilibrium) and $M = \frac{4\pi}{3} \rho R^3$

yields

$$L_{CO} \approx \sqrt{3\pi G/4\rho} T_{CO} M$$

The Mass-CO Luminosity Correlation

Solomon, Rivolo, Barrett & Yahil


The good correlation over 4 dex supported the assumption that GMCs are in virial equilibrium.

$M_{\text{virial}}$ may be an underestimate because it is based on optically thick CO. GMCs have diffuse regions that are not optically thick. And there are observational problems as well.
c. Correlations

The two observationally based correlations for GMCs are:
\[ \sigma = R^{1/2} \] (line width size relation)
\[ \frac{M}{R} \propto \sigma^2 \] (virial equilibrium)

Substitution leads to another
\[ N = \frac{M}{R^3} = \frac{\sigma^2}{R} \] (constant surface density)

and as well two more
\[ \rho = \frac{M}{R^3} = \frac{\sigma^2}{R^2} = \frac{1}{R} \quad \text{and} \quad M = \sigma^2 R = R^2 = \sigma^4 \]

The first three are often referred to as Larson’s Laws

Is it really true that the surface densities of GMCs are all about the same? Many have so assumed following Solomon et al.:
\[ N_H \sim 1.5 \times 10^{22} \text{ cm}^{-2} \]
\[ A_V \sim 10 \]
\[ \Sigma \sim 150 \ M_\odot \text{ pc}^{-2} \]

GMC Mass Spectrum


- The spectrum is incomplete for \( M < 10^5 \ M_{\odot} \) (dotted line).
- \( dN/dM \ M^{-3/2} \) for large \( M \)

To be addressed later:
1. What is the mass spectrum for clumps and cores?
2. How are cloud mass functions related to the stellar initial mass function (IMF)?
Reanalysis of Solomon et al. (1987)
Rosolowsky PASP 117 1403 2005

There is a sharp cutoff at $M = 3 \times 10^6 M_{\odot}$

Typical Properties of Local GMCs
Based on Solomon et al. (1987)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>4000</td>
</tr>
<tr>
<td>Mass</td>
<td>$2 \times 10^5 M_{\odot}$</td>
</tr>
<tr>
<td>Mean diameter</td>
<td>45 pc</td>
</tr>
<tr>
<td>Projected surface area</td>
<td>2000 pc$^2$</td>
</tr>
<tr>
<td>Volume</td>
<td>$10^5$ pc$^3$</td>
</tr>
<tr>
<td>Volume density ($H_2$)</td>
<td>300 cm$^{-3}$</td>
</tr>
<tr>
<td>Mean mass surface density</td>
<td>$1.5 \times 10^{22}$ cm$^{-2}$</td>
</tr>
<tr>
<td>Number surface density</td>
<td>4 kpc$^{-2}$</td>
</tr>
<tr>
<td>Mean separation</td>
<td>500 pc</td>
</tr>
</tbody>
</table>
Re-examining Larson’s Laws

Based on the UMASS-BU Galactic Ring Survey of $^{13}$CO (1-0)


Left side: UMass-BU $^{13}$CO (1-0) maps
Heyer et al 2009

Right side: UMass-SUNY $^{12}$CO (1-0)
defined GMCs (dashed lines)
Soloman et al 1987

New data:
- better resolution
- densely sampled
- more optically thin
- sensitive to lower $T_A$

NEW AND OLD GMC MASSES

Using Solomon definition of GMCs


New GMC masses are ~ factor of 5 smaller than the old virial theorem masses
NEW AND OLD CO LUMINOSITIES

New $L(\text{CO})$

Old $L(\text{CO})$

Heyer et al. (2009)

$\sim 50\%$ overestimate by Solomon et al. indicates that their extrapolation below the detectable brightness temperature over-estimates the luminosity.

Surface Density Distribution of GMCs

Heyer et al. (2009)

$1 \, M_\odot \, \text{pc}^{-2} \sim 10^{20} \, \text{H nuclei per sq cm}$

Not only is there is a factor of 5 difference in the medians, but GMCs do not all have the same surface densities.
Alternate Approach to Correlations

Start with the virial mass relation and the definition of surface density (with $N = \Sigma$), rather than with Larson’s law, and then:

substitute $N \approx \frac{M}{R^2}$ into $\frac{M}{R} \approx \sigma^2$ to get:

$$\sigma \approx N^{\frac{1}{2}} R^{\frac{3}{2}}$$

This indeed is what’s observed:

- open circles: $\Sigma$ from with cloud boundaries
- filled circles: “ 1/2 maximum isophote of $H_2$ column
- filled triangle: Solomon et al. (1987)

This is not the universal scaling law indicative of turbulence.

Similarity of the Extragalactic Correlation

Bolatto et al. IAU Symposium 255 274 2008
Understanding GMC Masses and Linewidths

1. Observe with better resolution, sampling, and sensitivity.
   See Goldsmith et al. ApJ 680 428 2008 for a 20", 32 pixel focal plane array study of the TMC, analyzed with a variable CO abundance model for diffuse regions. They obtain twice the mass compared to the fixed abundance model, with half the mass in diffuse regions.

2. Observe the HI with comparable resolution.

3. Observe and include magnetic fields and other measures of the velocity field in the analysis.

4. The origin of the supersonic linewidths seen in GMCs
   If it is not hydrodynamic turbulence, is it magnetic?
   • We show in Lecture 27 that the magnetic virial theorem gives $M \sim BR^2$ or $\Sigma \sim B$.
   • If the linewidths come from Alfven waves, $\sigma^2 \sim B^2/\rho$.
   • Replace $\rho$ by $M/R^3$ and use $M \sim BR^2$ to get $\sigma^2 \sim RB$, or $\sigma \sim \Sigma^{1/2} R^{1/2}$.

   This is Heyer’s result which he ascribes to Mouschovias (1987).

4. The CO / H₂ Conversion Factor

• Measuring the CO mass or column density is not the same as measuring the total gas, which is dominated by H₂ and He and are effectively invisible in cool clouds.
• The integrated CO intensity $I_{CO} = \int T_A(v) \, dv$ can be calibrated to yield the average H₂ column density. This is surprising because $^{12}$CO is optically thick and because the CO / H₂ ratio might be expected to vary within a cloud and from cloud to cloud.
• It is surprising that a single conversion factor between H₂ column density and $I_{CO}$ (the X-factor) applies on average to all molecular clouds in the Galaxy.
• That several calibration methods agree to within factors of a few should provide insights into the properties of the clouds.
X-factor Method 1: $I_{\text{CO}}$ and Virial Theorem

- Measured line intensity: $I_{\text{CO}} = I(^{12}\text{CO}) \equiv <T_A> \Delta v_{\text{FWHM}}$
- Virial theorem: 
  \[ \frac{GM}{R} = \sigma^2 = \left( \frac{\Delta v}{2.35} \right)^2 \]
- Mass estimate:
  \[ M = \frac{4\pi}{3} R^3 n(\text{H}_2) m \quad \text{and} \quad N(\text{H}_2) = \left( \frac{4\pi}{3} \right)^{-1} \frac{M}{m} \frac{R^2}{R^2} \]
- $\Delta v_{\text{FWHM}} = 2.35 \sigma \sim (GM/R)^{1/2}$

\[
\frac{N(\text{H}_2)}{I_{\text{CO}}} \approx 3 \times 10^{20} \text{ cm}^{-2} \text{ K}^{-1} \text{ km s}^{-1} \frac{10K}{T} \left( \frac{n(\text{H}_2)}{1000 \text{ cm}^{-3}} \right)^{1/2}
\]

**Problems:**
- Assumes virial equilibrium
- Depends on $n(\text{H}_2)$ and $T$
- Measures only mass within $\tau = 1$ surface

X-factor Method 2: $I_{\text{CO}}$ and NIR Extinction

- Measure $I_{\text{CO}}$ for regions with high $A_V$
- Determine $A_V$ from IR star counts
- Extrapolate $N_H/A_V$ from diffuse clouds
- Assume all hydrogen is molecular

Result:
\[ N(\text{H}_2) / I_{\text{CO}} \approx 4 \times 10^{20} \text{ cm}^{-2} / (\text{K km s}^{-1}) \]

**Problems:**
- Inaccuracies in star-count $A_V$
- Variable dust properties
- Variable $N_H/A_V$
- Best for dark clouds

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X-factor Method 3: $I(^{13}\text{CO})$ vs. $A_V$

- Determine $A_V$ as in method 2
- Measure $^{13}\text{CO}$ line intensity
- Assume $^{13}\text{CO}$ optically thin, $^{12}\text{CO}$ optically thick
- Assume $T_{\text{ex}}(^{13}\text{CO}) = T_{\text{ex}}(^{12}\text{CO})$
- Assume $^{12}\text{CO} / ^{13}\text{CO} = 40 \ldots 60 \Rightarrow \tau(^{13}\text{CO}) \Rightarrow N(^{13}\text{CO})$

Problems:
- Accuracy of $A_V$ determination
- Often $T_{\text{ex}}(^{13}\text{CO}) < T_{\text{ex}}(^{12}\text{CO})$
- $^{13}\text{CO}$ may not be optically thin

X-factor Method 4: $I_{\text{CO}}$ and $\gamma$-Rays

- High energy comic rays (> 1 GeV) produce neutral pions in collisions with protons in H and $H_2$, which then decay into two $\gamma$-rays
  \[ p + p \rightarrow p' + p' + \pi_0, \quad \pi_0 \rightarrow \gamma + \gamma \]
- The $\gamma$-ray emission depends on the product of the cosmic ray density and the density of all protons ($n_H$).
- Hunter et al. ApJ 481 205 1997 combine $\gamma$-ray measurements from COMPTON/EGRET with the Columbia-CfA CO survey and obtain,
  \[ N(H_2)/I_{\text{CO}} = (1.56 \pm 0.05) \times 10^{20} \text{ cm}^{-2} \text{ K}^{-1} \text{ km}^{-1}. \]

presumably assuming all hydrogen is molecular.

**NB** The modulation correction for high energy CRs is small.
Hunter et al. assume that the CR density is proportional to $n_H$. 

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X-factor Method 5: HI/IRAS/CO

- Dame et al. (ApJ 547 792 2001) used IRAS far–IR emission as a tracer of total gas column density
- Calibrated with the Leiden–Dwingeloo 21–cm HI survey in regions free of CO emission
- Total gas map differenced with the HI map to obtain a complete and unbiased predicted map of H$_2$
  - Close agreement between this map and observed CO implies that few molecular clouds at |b| < 30° have been missed by CO surveys
- The ratio of the observed CO map to the predicted molecular map provides a measure of the local average X–factor for |b| > 5°:

$$N(H_2)/I_{CO} = 1.8 \pm 0.3 \times 10^{20} \text{ cm}^{-2} \text{ K}^{-1} / \text{km s}^{-1}$$
Method 5: HI/IRAS/CO

Dame et al. compared IRAS far-IR (dust), 21 cm (HI) and 2.6 mm (CO).

Verification of Method 5
James Graham’s Critique of Method 5

- Average X varies with latitude
  - High X at l ~ 0° may be spurious since, the lack of CO-free regions toward the inner plane mean $h_{tot}/N_{tot}$ cannot be properly determined
- Point-to-point dispersion is significantly larger than can be accounted for by instrumental noise
  - Excluding the plane ($|b| < 5°$), where the prediction is expected to break down owing to dust temperature variations along the line of sight, the dispersion is ~ 50%
  - The high dispersion may be due to variations in the gas-to-dust ratio, and by dust temperature variations not accounted for by the simple IRAS color correction

C.f. JRG Lecture 18 (2006)

CO/H$_2$ Conversion Factors: Summary

- Various methods agree remarkably well
- Relevant on global scales, not locally
- Limits on applicability are unclear
- No information on $N(H_2) / N(CO)$ is obtained
- Conversion factors should depend on $T$, $n$ and metallicity
- Conversion factor derived for Milky Way disk is not valid for galactic nuclei (including our own Galactic Center) or for metal-poor systems
- Blitz et al. (PPV) find that $X_{CO} \approx 4 \times 10^{20}$ cm$^{-2}$ (K km s$^{-1}$)$^{-1}$ holds approximately for members of the local group, but not the SMC, where $X_{CO} \approx 13.5 \times 10^{20}$ cm$^{-2}$ (K km s$^{-1}$)$^{-1}$. The conversion for the LMC, $X_{CO} \approx 9.0 \times 10^{20}$ cm$^{-2}$ (K km s$^{-1}$)$^{-1}$, also reflects the reduced abundances of the clouds.
## CO/H\(_2\) Conversion Factor: Summary

<table>
<thead>
<tr>
<th>Source</th>
<th>(\chi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early work</td>
<td>2-5</td>
</tr>
<tr>
<td>(\gamma)-rays (Hunter et al. 1997)</td>
<td>1.56 ±0.05</td>
</tr>
<tr>
<td>HI/IRAS/CO (Dame et al. 2001)</td>
<td>1.8±0.3</td>
</tr>
<tr>
<td>IR extinction (Lada et al. 2003)</td>
<td>(~ 4)</td>
</tr>
</tbody>
</table>

Units for \(\chi\): \(10^{20} \text{ cm}^{-2} / \text{ K km s}^{-1}\)