Astronomy C10
Quiz 2 Solutions

Sections 102, 111, 122, 132

October 26-30, 2020

This quiz is worth a total of 50 points, and you will have 20 minutes. There are 3 problems with a few parts each. I would suggest reading over the questions first, and then starting on the topics with which you are most comfortable. Please show your work/write your thoughts so that I can award partial credit! All the math can be done with ratios – no calculator needed.

Good luck!

Some useful equations and constants:

$h = 6.6 \times 10^{-34}\,\mathrm{J\,s}$	$E = hf = \frac{hc}{\lambda}$	$1\,\mathrm{m} = 10^9\,\mathrm{nm} = 10^{10}\,\text{Å}$
$c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$	$L = 4\pi R^2 \sigma T^4$	$\lambda T = 2.9 \times 10^{-3}$ Kelvin-meters
$c = \lambda f$	$\dfrac{\lambda - \lambda_0}{\lambda_0} = \dfrac{v}{c}$	$P^2 = ka^3, k = \frac{4\pi^2}{G(m_1+m_2)}$
$b = L/(4\pi d^2)$	$L \propto M^4$	$t_{\mathrm{lifetime}} \propto M/L$

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1 Question 1: 51-Pegasi b (20 points)

In October of 1995, astronomers announced the discovery of the first exoplanet, 51-Pegasi b. Astronomers at the University of Geneva used the radial velocity ("Doppler Wobble" method to infer the exoplanet's existence, so let's take a look at a similar system for this problem.

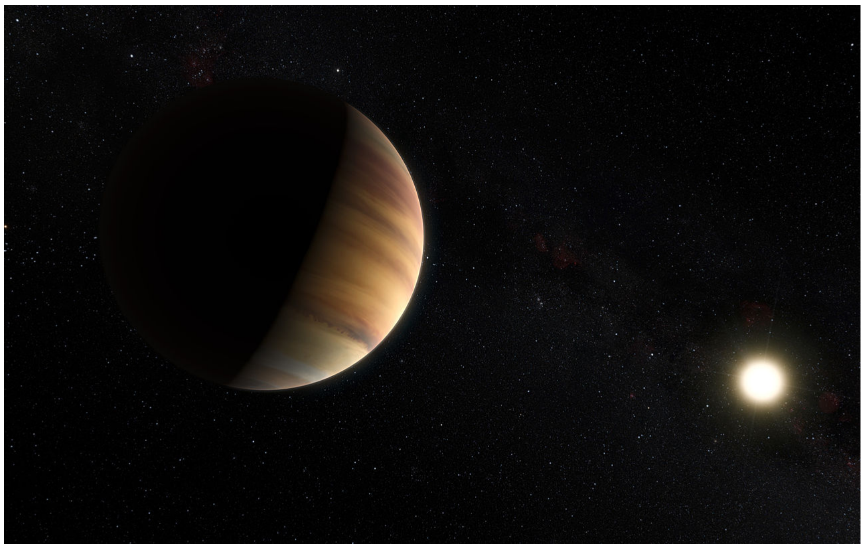

Figure 1: Artist impression of what 51-Peg b might look like. Taken from Wikipedia.

(A) Describe the radial velocity ("Doppler Wobble") method and how we use this method to detect exoplanets. What is the main limitation of the "Doppler Wobble" method? (5 points)

The radial velocity method measures the Doppler shift of a planets spectrum to infer the existence of an exoplanet. An exoplanet and its host star orbit the center of mass of the system, leading to slight motion of the host star. This motion is detected via Doppler shifts, allowing us to infer that an exoplanet exists.

The limitation is that we cannot know the inclination of the orbit. Thus, we only measure the **minimum mass** of the exoplanet. (Points were also given for mentioning long period orbits).

(B) Describe the transit method of exoplanet detection. What, specifically, can we infer from measurements of an exoplanet transit? (5 points)

We use the transmit method by measuring the brightness of a star as a function of time. As an exoplanet passes in front of the star (along our line of sight), it slightly dims the host star. Observing many of these dimming events allows us to infer the existence of an exoplanet. The "dip" in the brightness allows us to calculate the **radius** of the exoplanet. Points were also given for period of orbit.

(C) Let's say that 51-Pegasi b was orbiting a star with a mass $M = 4M_{\text{sun}}$. Additionally, let's say that 51 Pegasi-b has highly eccentric orbit ($e = 0.75$) and has a semi-major axis of $a = 4$ AU. What is the period of 51-Pegasi b's orbit? (6 points)

The eccentricity is irrelevant here – Kepler's 3rd Law only rely on the semi-major axis. It's probably easiest to use this form:

$$P^2\,[\text{years}] = \frac{a^3\,[\text{AU}]}{M\,[M_\odot]}$$

remembering that Kepler's "constant" k is not a constant but actually depends on the mass of the host star. Pluggingin the given numbers, we have:

$$P^2\,[\text{years}] = \frac{4^3\,[\text{AU}]}{4\,[M_\odot]} = 4^2 \rightarrow P^2 = 16 \text{ years} \rightarrow \boxed{P = 4 \text{ years}}$$

(D) Let's say there is an alien species that is equidistant (exactly in between the two; meaning the distance to Earth is the same as the distance to 51-Pegasi b) from Earth and 51-Pegasi b. Knowing that 51-Pegasi b is much more massive than Earth and closer to its host star than Earth is to the Sun, which system – Earth-Sun or 51-Pegasi b and its host star – causes a more pronounced "Doppler Wobble", as measured by the aliens? (4 points)

The fundamental reason the host star moves is due to the force of gravity:

$$F_{gravity} = \frac{GM_{star}M_{planet}}{d^2}$$

where the d is the distance between the planet and the star. 51-Pegasi b has a smaller d and larger M_{planet} (and larger M_{star}) so the gravitational pull of 51-Peg b will be stronger. Therefore the Doppler wobble is more pronounced for 51-Pegasi b.

2 Question 2: Stellar Evolution (18 points)

As we learned in class, a stellar spectrum is composed of a blackbody continuum with absorption features superimposed according to the chemical makeup of the star. Let's explore this idea and see what information we can learn about stars. Consider two blackbody curves, one representing Star A (blue) and one representing Star B (red).

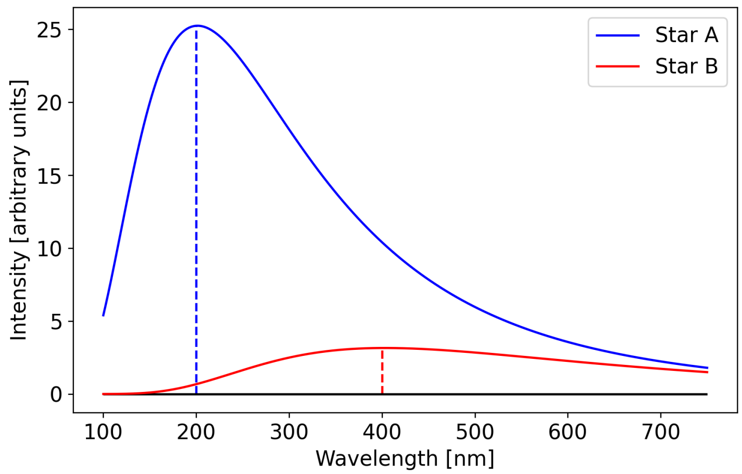

Figure 2: Blackbody curves for Problem 2.

(A) Star B is entering the Red Giant phase and is therefore $4\times$ larger in radius than Star A. Given this and the plot above, which star is more luminous and by what factor? (6 points)

We need to know the ratio of both the radii and the temperatures to answer this question since $L = 4\pi R^2 \sigma T^4$. We are given the ratio of the radii (4), and we can get the ratio of the temperatures from the graph of the spectra by applying Wien's Law. We find:

$$\frac{T_A}{T_B} = \frac{\lambda_{peak,B}}{\lambda_{peak,A}} = 2$$

We then can apply the luminosity equation:

$$\frac{L_A}{L_B} = \left(\frac{R_A}{R_B}\right)^2 \left(\frac{T_A}{T_B}\right)^4 = \left(\frac{1}{4}\right)^2 (2)^4 = 1$$

They are the same luminosity.

(B) Using the answer to (A), and knowing that Star A is 2× as far as Star B, which star appears brighter and by what factor? (6 points)

We know that $b = \frac{L}{4\pi d^2}$ where d is the distance to the star. Solving this with ratios (and knowing from (a) that they are the same luminosity – though, I didn't take off if you used your answer from (a)):

$$\frac{b_A}{b_B} = \frac{L_A}{4\pi d_A^2}\frac{4\pi d_B^2}{L_B} = \frac{d_B^2}{d_A^2} = 2^2 = 4$$

Star A is thus 4 times as bright as Star B.

(C) Let's say that Star A is on the Main Sequence, and as we a said above, Star B is a red giant. What are **two intrinsic differences** (i.e., composition, power generation/fusion properties, HR diagram, etc.) between these two stars, knowing this information? (6 points)

There are a variety of answers for this question. You can see Course Slides 167-177 for more information.

Some of the answers include that Main Sequence stars fuse hydrogen to helium in the core, whereas red giants fuse Helium into Carbon and Oxygen in the core. Red Giants are more evolved types of stars, whereas Main Sequence stars are in the "prime" of their life. Red Giants are typically cooler than most main sequence stars, but they are extremely luminous because of their large radii. Red Giants are found on the upper-right of the HR diagram, at high luminosity but low temperature.

3 Question 3: Supernovae and Stellar Death (12 points)

A standard picture for a Type Ia supernova can be seen below. Consider a white dwarf-red giant binary star system that is on the verge of having a nova event. The white dwarf is accreting mass from the red giant. This question concerns this system.

Figure 3: Artist impression of a white dwarf-red giant binary star system. The white dwarf is on the left.

(A) The white dwarf has a mass $M_{\text{white dwarf}} = 1M_{\text{sun}}$ and the red giant has a mass $M_{\text{red giant}} = 3M_{\text{sun}}$. The semi-major axis of the white dwarf's orbit is 2 AU. What is the orbital period of this system? (5 points)

We need to use Newton's version of Kepler's 3rd Law here since neither object has negligible mass. Recall that Newton's version:

$$P^2 = \frac{4\pi^2}{G\left(M_1 + M_2\right) a^3}$$

We always should pick units that are convenient, meaning we should scale this to the Solar System:

$$P^2\,[\text{years}] = \frac{a^3\,[\text{AU}]}{\left(M_1 + M_2\right)[M_\odot]}$$

We can essentially plug in numbers now:

$$P^2\,[\text{years}] = \frac{2^3\,[\text{AU}]}{(1+3)\,[M_\odot]} = 2 \rightarrow \boxed{P = \sqrt{2}\text{ years}}$$

(B) How does the concept of a "Roche lobe" apply to this system and accretion (mass-transfer) between the two stars? Write one sentence. (3 points)

The Roche Lobe is the region around a star inside of which the gravity of that star dominates. If material from a star exceeds the Roche Lobe boundary, mass transfer (accretion) starts.

Any explanation of the above received credit.

(C) Let's now consider a different system with two stars, Star A and Star B, both on the Main Sequence. They have masses $M_\text{A} = 1M_\text{sun}$ and $M_\text{B} = 2M_\text{sun}$. Which star is more luminous assuming everything else is equal? It might help to think of the HR diagram. (4 points)

One of the key relationships we have learned is $L \propto M^4$ for stars on the Main Sequence. If we increase the mass, we increase the luminosity, so whichever star is more massive will be more luminous assuming everything else is equal. In this case, M_B is more massive.