On the migration of a system of protoplanets

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ABSTRACT
The evolution of a system consisting of a protoplanetary disc with two embedded Jupiter-sized planets is studied numerically. The disc is assumed to be flat and non-self-gravitating; this is modelled by the planar (two-dimensional) Navier–Stokes equations. The mutual gravitational interaction of the planets and the star, and the gravitational torques of the disc acting on the planets and the central star are included. The planets have an initial mass of one Jupiter mass $M_{\text{Jup}}$, each, and the radial distances from the star are one and two semimajor axes of Jupiter, respectively.

During the evolution a joint wide annular gap is created by the planets. Both planets increase their mass owing to accretion of gas from the disc: after about 2500 orbital periods of the inner planet it has reached a mass of $2.3 M_{\text{Jup}}$, while the outer planet has reached a mass of $3.2 M_{\text{Jup}}$. The net gravitational torques exerted by the disc on the planets result in an inward migration of the outer planet on time-scales comparable to the viscous evolution time of the disc. The semimajor axis of the inner planet remains constant as there is very little gas left in its vicinity to induce any migration. When the distance of close approach eventually becomes smaller than the mutual Hill radius, the eccentricities increase strongly and the system may become unstable.

If disc depletion occurs rapidly enough before the planets come too close to each other, a stable system similar to our own Solar system may remain. Otherwise the orbits may become unstable and produce systems like $\upsilon$ And.

Key words: accretion, accretion discs – hydrodynamics – planets and satellites: general – stars: formation – planetary systems.

1 INTRODUCTION
It is generally assumed that planetary systems form in a differentially rotating gaseous disc. In the late stages of their formation, the protoplanets are still embedded in the protostellar disc and their orbital evolution is coupled to that of the disc. Gravitational interaction between the planets and the gaseous disc has basically two effects.

(i) The torques by the planets acting on the disc tend to push away material from the orbital radius of the planet, and for sufficiently massive planets a gap is formed in the disc (Papaloizou & Lin 1984; Lin & Papaloizou 1993; Takeuchi, Miyama & Lin 1996). The dynamical process of gap formation has been studied through time-dependent hydrodynamical simulations for planets on circular orbits by Bryden et al. (1999) and Kley (1999, hereafter Paper I), and more recently by Lubow, Seibert & Artymowicz (1999).

(ii) Additionally, the gravitational force exerted by the disc alters the orbital parameter (semimajor axis, eccentricity) of the planet. Here, these forces typically induce some inward migration of the planet (Goldreich & Tremaine 1980) which is coupled to the viscous evolution of the disc (Lin & Papaloizou 1986). A two-dimensional computation of the migration of an embedded planet in an inviscid disc has been presented by Nelson & Benz (1999). Hence the present location of the observed planets (solar and extrasolar) may not be identical to the position at which they formed.

The results indicate that, even after the formation of a gap, the planet may still accrete material from the inner and outer disc and reach about 10 Jupiter masses ($M_{\text{Jup}}$). For very low disc viscosity and larger planetary masses the mass accumulation finally terminates (Bryden et al. 1999; Lubow et al. 1999).

In particular, the migration scenario applies to some of the extrasolar planets (for a summary of their properties see Marcy, Cochran & Mayor 1999): the 51 Peg-type planets. They all have masses of the order of $M_{\text{Jup}}$, and orbit their central stars very closely, having orbital periods of only a few days. As massive planets, according to standard theory, have formed at distances of
a few au from their stars, these planets must have migrated to their present position. The inward migration was eventually halted by tidal interaction with the star or through interaction with the stellar magnetosphere (Lin, Bodenheimer & Richardson 1996). The only extrasolar planetary system around a main-sequence star known so far (υ And) consists of one planet at 0.059 au on a nearly circular orbit and two planets at 0.83 and 2.5 au having larger eccentricities (0.18 and 0.41) (Butler et al. 1999).

In the case of the Solar system, the question arises of what prevented any further inward migration of Jupiter. As the net tidal torque on the planet is a delicate balance between the torque of the material located outside the planet and that of the material inside (e.g. Ward 1997), any perturbation in the density distribution material located outside the planet and that of the material inside prevented any further inward migration of Jupiter. As the net tidal torques (0.18 and 0.41) (Butler et al. 1999).

As the orbital range covers a complete ring of the (moving) star. Thus, in addition to the gravitational potential (1), the disc and planets feel the additional acceleration \( -a_c \).

The mass accreted from the disc by the planets (see below) has some net angular momentum which in principle also changes the orbital parameter of the planets. However, this contribution is typically about an order of magnitude smaller than the tidal torque (Lin et al. 2000) and is neglected here.

As the details of the origin and magnitude of the viscosity in discs are still uncertain, we assume a Reynolds stress formulation (Paper I) with a constant kinematic viscosity. The temperature distribution of the disc is fixed throughout the computation and is given by the assumption of a constant ratio of the vertical thickness \( H \) and the radius \( r \). Hence the fixed temperature profile is given by \( T(r) \propto r^{-1} \). We assume \( H/r = 0.05 \), which is equivalent to a fixed Mach number of 20.

For numerical convenience we introduce dimensionless units, in which the unit of length, \( R_0 \), is given by the initial distance of the first planet to the star \( R_0 = r_1(t = 0) = 1a_{Jup} \). The unit of time is obtained from the (initial) orbital angular frequency \( \Omega_1 \) of the first planet, i.e. the orbital period of planet 1 is given by

\[
P_1 = 2\pi \Omega_1 \tag{3}
\]

The evolutionary time of the results of the calculations as given below will usually be stated in units of \( P_1 \). The unit of velocity is then given by \( v_0 = R_0/\Omega_1 \). The unit of the kinematic viscosity coefficient is given by \( \nu_0 = R_0 v_0 \). Here we take a typical dimensionless value of \( 10^{-5} \) corresponding to an effective \( \alpha \) of \( 4 \times 10^{-3} \).

2.1 The numerical method in brief

The normalized equations of motion are solved using an Eulerian finite difference scheme, where the computational domain \([r_{\text{min}}, r_{\text{max}}] \times [\varphi_{\text{min}}, \varphi_{\text{max}}]\) is subdivided into \( N_r \times N_\varphi \) grid cells. For the typical runs we use \( N_r = 128 \) and \( N_\varphi = 128 \), where the azimuthal spacing is equidistant, and the radial points have a closer spacing near the inner radius. The numerical method is based on a spatially second-order-accurate upwind scheme (monotonic transport), which uses a formally first-order time-stepping procedure. The methodology of the finite difference method for disc calculations is outlined in Kley (1989) and Paper I.

The \( N \)-body module of the program uses a fourth-order Runge-Kutta method for the integration of the equations of motion. It has been tested for long-term integrations studying the onset of instability in the three-body problem consisting of two closely spaced planets orbiting a star, as described by Gladman (1993). For the initial parameter used here, the error in the total energy after \( 1.2 \times 10^5 \) orbits (integration over \( 10^8 \) yr) is less than \( 2 \times 10^{-9} \).

2.2 Boundary and initial conditions

To cover fully the range of influence of the planet on the disc, we typically choose for the radial extent (in dimensionless units, where planet 1 is located initially at \( r = 1 \)) \( r_{\text{min}} = 0.25 \) and \( r_{\text{max}} = 4.0 \). The azimuthal range covers a complete ring \( \varphi_{\text{min}} = 0.0 \), \( \varphi_{\text{max}} = 2\pi \) using periodic boundary conditions. To test the accuracy of the migration, a comparison calculation with \( r_{\text{max}} = 8.0 \) and higher resolution \( N_r = 256 \), \( N_\varphi = 256 \) has also been performed.

The outer radial boundary is closed to any mass flow \( \varnothing(r_{\text{max}}) = 0 \), while at the inner boundary mass outflow is allowed, simulating
accretion on to the central star. At the inner and outer boundaries
the angular velocity is set to the value of the unperturbed
Keplerian disc. Initially, the matter in the domain is distributed
independently of the viscosity in the disc, and for the given value of
$D$ for this ‘gap clearing’ process can be estimated by
time that it takes to transport material with a given
angular momentum a certain distance $\Delta$ given the torque. Bryden et al. (1999) estimate this time to be of the order of

$$t_d = \frac{1}{q^2} \left( \frac{\Delta}{r_0} \right)^5 P_n,$$

(4)

where $r_0$ is the location of the planet having mass ratio $q$ that
orbits the star with a period $P$. For a typical half-width of the gap
$\Delta = 0.2r_0$ and a mass ratio $q = 0.001$, one finds for the clearing
time $t_d = 320P$. The time to clear the gap as inferred from Fig. 1
agrees very well with this estimate. As has been seen already in
previous investigations, the gap formation in an otherwise
unperturbed initial disc begins after only a few orbital periods
(Bryden et al. 1999; Paper I). This initial clearing time is
independent of the viscosity in the disc, and for the given value of
$q$ this time is much smaller than the typical viscous time-scale $t_\nu$.

Similarly to the one-planet calculations described in Bryden et al. (1999), Paper I and Lubow et al. (1999), each of the two
planets creates a spiral wave pattern (trailing shocks) in the
density of the disc. In the case of one disturber on a circular orbit,
the pattern is stationary in the frame corotating with the planet.
The presence of a second planet makes the spirals non-stationary,
as is seen in the snapshots after 50, 100, 250 and 500 orbits of the
inner planet that are displayed in Fig. 2. Near the outer boundary
at $r = 4$ the reflection of the spiral waves is visible. Using the
higher resolution model with $r_{\text{max}} = 8.0$ (Section 2.2), we have
tested whether the numerical resolution or the wave reflection at
$r_{\text{max}}$ has any influence on the calculation of the net torques acting
on the two planets and the accretion rates on to them. Owing to
limited computational resources, the higher resolution model was
run only for a few hundred orbital periods, and the largest
difference (2.5 per cent) occurred in the mass $m_3$ of the outer
planet. The difference in radial position (migration) is less than
1 per cent. We may conclude that our resolution was chosen to be
sufficiently fine, and that the reflections at the outer boundary
$r_{\text{max}} = 4$ do not change our conclusions significantly.

In previous calculations (Paper I) the equilibrium mass
accretion rate from the outer part of the disc on to a $1 M_{\text{jup}}$
planet for the same viscosity ($\nu = 10^{-5}$) and distance from the star
was found to be $4.35 \times 10^{-6} M_{\text{jup}} \text{yr}^{-1}$ for a fully developed gap.
Here the accretion rate on to the planets is much higher in the
beginning ($=5 \times 10^{-4} M_{\text{jup}} \text{yr}^{-1}$) as the initial gap was not as well
cleared. Thus, during this gap clearing process, the masses of the
individual planets grow rapidly at the onset of the calculations
(Fig. 3). At $t = 250$ the mass within the the gap has been
exhausted (see also Fig. 1) and the accretion rates $M$ on to the
planets lower dramatically. At later times, after several hundred
orbits ($P_1$), they settle to nearly constant values of about
$10^{-6} M_{\text{jup}} \text{yr}^{-1}$ for the outer planet and $2.2 \times 10^{-6} M_{\text{jup}} \text{yr}^{-1}$
for the inner planet (from Fig. 3). Since the mass inside planet 1 has
left the computational domain and the initial mass between the
two planets has been consumed by the two planets, this mass
accretion rate on to planet 1 for later times represents the mass
flow of material coming from radii larger than $r_2$ (beyond the
outer planet). It is the material that has been flowing across the
gap of the outer planet. Previously, this mass flow across a gap has
been calculated to be about $1/7$ of the mass accretion rate on to a
planet (Paper I), and the present results are entirely consistent with
that estimate.

The gravitational torques exerted by the disc lead to an

\begin{center}
Figure 1. The azimuthally averaged surface density for different times
(stated in units of $P_1$). The solid line refers to the initial density
distribution. The density inside planet 1 is reduced because of mass leaving
through $r_{\text{min}}$.\end{center}
additional acceleration of the planets, resulting in an expression similar to the acceleration of the star (equation 2). For one individual planet this force typically results in an inward migration of the planet on time-scales of the order of the viscous time of the disc (Lin & Papaloizou 1986). Through their detailed two-dimensional computations, Nelson et al. (2000) estimate this migration time to be of the order of $10^4$ initial orbits of the planet.

Here, this inward migration (with a similar time-scale) is seen clearly for the outer planet in Fig. 4, where the time evolution of the semimajor axes of the two planets is plotted.

The inner planet, on the other hand, initially for $t < 200$ moves slightly inwards, but then the semimajor axis increases and,
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4 CONCLUSIONS

We have presented calculations of the long-term evolution of two embedded planets in a protoplanetary disc that cover several thousand orbital periods. The planets were initially located at $1a_{\text{sup}}$ and $2a_{\text{sup}}$ from the central star with initial masses of $1M_{\text{Jup}}$ each. The gravitational interaction with the gaseous disc, having a total mass of 0.01 $M_{\odot}$, leads to an inward migration of the outer planet, while the semimajor axis of the inner planet remains approximately constant and even increases slightly. At the same time, the ongoing accretion on to the planets increases their masses continuously until, at the end of the simulation (at $t = 2500$), the outer planet has reached a mass of about $3.2M_{\text{Jup}}$ and the inner planet has reached a mass of about $2.3M_{\text{Jup}}$. This increase in mass and the decreasing distance between the planets eventually causes the orbits to become unstable, resulting in a strong increase of the eccentricities on time-scales of a few hundred orbits.

From the computations we may draw three major conclusions.

(i) The inward migration of planets immersed in an accretion disc may be halted by the presence of additional protoplanets located, for example, at larger radii. After having grown sufficiently in mass, they disturb the density distribution significantly by forming, for example, one large joint gap. This in turn reduces the net gravitational torque acting on the inner planet. Thus the migration of the inner planet is halted, and its semimajor axis remains nearly constant.

(ii) When disc depletion occurs sufficiently rapidly to prevent a large inward migration of the outer planet(s), a planetary system with massive planets at a distance of several au remains. This scenario may explain why, for example, in the Solar system the outer planets (in particular Jupiter) have not migrated any closer to the Sun.

(iii) If the initial mass contained in the disc is sufficiently large then the inward migration of the outer planet(s) will continue until some of them reach very close spatial separations. This will lead to unstable orbits, resulting in a strong increase of the eccentricities. Orbits may cross, and then planets either may be driven to highly eccentric orbits or may leave the system altogether (see e.g. Weidenschilling & Marzari 1996). This may then explain the high eccentricities in some of the observed extrasolar planets, in particular the planetary system of v Andromedae.

As planetary systems containing more than two planets display different stability properties (Chambers et al. 1996), it will be interesting to study the evolution of multiple embryos in the protoplanetary nebula.

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