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Light-scattering models applied to circumstellar dust properties

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Abstract

Radiation pressure force, Poynting–Robertson effect, and collisions are important to determine the size distribution of dust in circumstellar debris disks with the two former parameters depending on the light-scattering properties of grains. We here present Mie and discrete-dipole approximation (DDA) calculations to describe the optical properties of dust particles around β Pictoris, Vega, and Fomalhaut in order to study the influence of the radiation pressure force. We find that the differences between Mie and DDA calculations are lower than 30% for all porosities. Therefore, Mie calculations can be used to determine the cut-off limits which contribute to the size distribution for the different systems.

Keywords: Circumstellar dust; Light scattering; Radiation pressure; Porous particles

1. Introduction

While the existence of dust around young stars as remnants of the star formation has been known and studied for a long time, improved infrared observations have revealed the existence of dust around evolved stars that are close to or within their main-sequence stage. Estimates of the dust lifetimes in these systems indicate that the dust cannot be primordial, but has to be produced after the formation of the star. The spatial distribution is explained by the presence of planetesimals that are also assumed as a source of the dust [1]. These dust shells are referred to as second-generation dust clouds or circumstellar debris disks and most likely are part of planetary systems in an early evolutionary stage. Light-scattering models are an important tool for analyzing observational data and for studying the circumstellar dust properties.

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An important piece of information for understanding the evolution of these circumstellar planetary systems is the total amount of material in the system which critically depends on the size distribution of dust. There are several sets of size distributions that can fit the observational data and, therefore, we need more information about the behavior of the particles in the dust disk to further constrain the size distribution of the particles [2]. The radiation pressure force acting on particles in bound Keplerian orbits causes a small deceleration, known as the Poynting–Robertson effect, so that finally the particles fall into the star if they are not destroyed by collisions. Hence the radiation pressure determines the Poynting–Robertson lifetime of the particles. It also determines the size limit of particles that are ejected from the system in hyperbolic orbits when radiation pressure exceeds gravitational attraction. These parameters, together with the collision lifetime constrain the size distribution in the system. Therefore, the knowledge of radiation pressure force is crucial for a better understanding of the grain size distribution. In this paper, we calculate radiation pressure forces on porous and fluffy grains with Mie theory and discrete-dipole approximation (DDA) and discuss their dynamical consequences.

2. Calculation methods

2.1. Forces acting on the particles

The influence of the radiation pressure force is discussed as the ratio of radiation pressure force to gravitational force

$$\beta = \frac{F_{\mathrm{rp}}}{F_{\mathrm{grav}}} \tag{1}$$

and calculated as

$$\beta = \frac{\pi R_\star^2}{\gamma M_\star c m} \int_0^\infty B_\star(\lambda, T) C_{\mathrm{pr}}(s, \lambda)\, \mathrm{d}\lambda, \tag{2}$$

where $M_\star$ is the mass of the star, $R_\star$ the radius of the star, γ the gravitational constant, c the speed of light, m the mass of a grain, $B_\star$ the Planck function, T the stellar photospheric temperature, C_{pr} the cross section for the radiation pressure, s the particle size, and λ the wavelength [3]. We study a selected sample of stars where the observed infrared-excess is (partly) attributed to a circumstellar debris disk and where also some spatially resolved observations were already made. The stars we consider are Vega, β Pictoris, and Fomalhaut.

The cross sections for the radiation pressure C_{pr} is calculated as

$$C_{\mathrm{pr}} = C_{\mathrm{abs}} + C_{\mathrm{sca}}(1 - g), \tag{3}$$

where C_{abs} is the absorption cross section, C_{sca} the scattering cross section and g the asymmetry parameter [4]. The cross section C_{pr} then depends on the absorption and scattering properties of grains and can be calculated with light-scattering theories. We here compare DDA and Mie theory.

Mie theory describes the scattering and absorption properties of compact spherical particles by solving the Maxwell equations with given boundary conditions. The Mie calculations are carried out with a code given in Bohren and Huffmann [4]. The input parameters are the refractive indices, the size of grains, and the wavelength of scattered light.

Table 1
Number of dipoles and volume-equivalent radia

Vacuum fraction	0%	30%	60%	90%
Number of dipoles	15 515	10 925	6161	1475
Volume-equivalent radius	0.05	0.044	0.037	0.023
	0.07	0.062	0.052	0.033
	0.1	0.089	0.074	0.046
	0.2	0.178	0.147	0.093
	0.3	0.266	0.221	0.139

Calculating porous particles the refractive indices are determined for different porosities of a given material using the Maxwell–Garnett mixing rule. The complex dielectric function $\varepsilon = \varepsilon' + i\varepsilon''$ is derived from the complex refractive index $m = n + ik$ by $\varepsilon' = n^2 - k^2$ and $\varepsilon'' = 2nk$. The Maxwell–Garnett theory then describes the complex dielectric function $\varepsilon_{\rm av}$ of a grain that consists of a homogeneous matrix material $\varepsilon_{\rm m}$ and inclusion material ε as

$$\varepsilon_{\rm av} = \varepsilon_{\rm m}\left[1 + \frac{3f\left(\frac{\varepsilon-\varepsilon_{\rm m}}{\varepsilon+2\varepsilon_{\rm m}}\right)}{1 - f\left(\frac{\varepsilon-\varepsilon_{\rm m}}{\varepsilon+2\varepsilon_{\rm m}}\right)}\right], \tag{4}$$

where *f* is the volume fraction of inclusions. Mie calculations are then carried out with the new complex refractive index calculated from Maxwell–Garnett mixing rule to obtain the scattering and absorption cross sections of a spherical porous particle. For porous particles the inclusions are assumed as vacuum. This approach is valid if the size of the inclusions is small compared to the wavelength of scattered light. While the Mie theory gives the exact description of the light scattering of compact spheres we will check to what extent it applies to porous particles by comparing to DDA calculations.

The DDA code is developed by Draine and Flatau [5]. It describes the particles as agglomerates of discrete interacting dipoles to calculate the scattering and absorption properties. The spatial configuration of the dipoles can describe particles of arbitrary shape. Conditions for the applicability are a small size parameter of dipoles $x = \frac{2\pi a}{\lambda} < 1$, where *a* is the dipole size and λ the wavelength, and a refractive index *m* with: $|m - 1| < 1$.

We here use DDA to calculate the scattering of compact and porous spherical particles in the wavelength range from 0.14 to 300 μm. We place the particle in the center of a box of $31 \times 31 \times 31$ dipoles and within that define a sphere with radius of 15 dipoles so that it has 15 515 dipoles. For the case of the highest porosity this provides the minimum number of dipoles for which convergence was achieved. The radii *s* of the considered compact spheres are 0.05, 0.07, 0.1, 0.2, and 0.3 μm. To generate porous particles dipoles are randomly taken out of the initial sphere, as described in [6]. The volume equivalent radii are determined by

$$s_{ve} = s \cdot (1 - p)^{1/3}, \tag{5}$$

where *p* is the porosity (fraction of vacuum inclusions) and *s* is the radius of compact spherical grains. The respective values of the number of dipoles and the volume-equivalent radii of a sphere are listed in Table 1.

The orientations of a spherical particle do not play an important role for the scattering and absorption properties so we determine three angles for each of the three particle orientations β, θ, and ϕ for spherical

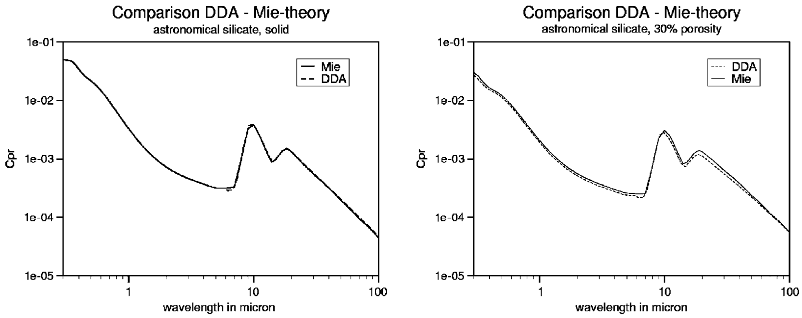

Fig. 1. C_{pr}-calculations with Mie theory and DDA for astronomical silicate for compact (left) and fluffy spherical particles with a porosity of 30% (right).

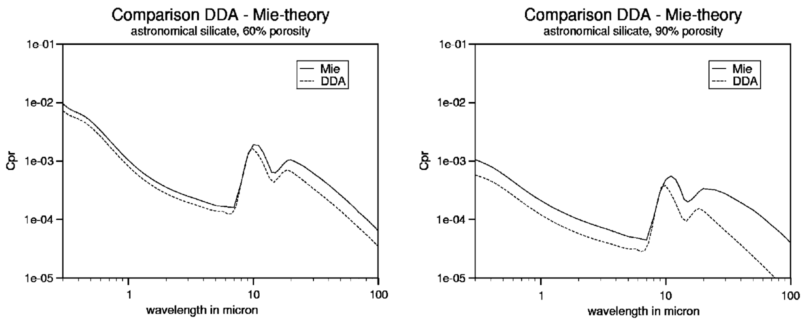

Fig. 2. C_{pr}-calculations with Mie theory and DDA for astronomical silicate for porous particles with 60% (left) and 90% porosity (right).

compact grains and particles with porosities of 30% and 60%. For a porosity of 90% the orientations are increased to seven values of each β, θ, and ϕ because the particles have an irregular surface and open structure. We take the optical constants for astronomical silicate, a material mixed of crystalline olivine and amorphous silicate with an olivine normative composition which is only slightly absorbing [7]. As an example for a strongly absorbing material we consider amorphous carbon [8].

3. Results and discussion

In Figs. 1 and 2 the cross section of radiation pressure C_{pr} is plotted versus wavelength for different porosities for astronomical silicate particles. We consider porosities of 30%, 60% and 90% for both theories. The figures show that calculations with Mie theory and DDA give nearly the same values of C_{pr} for compact spherical particles, with differences for wavelengths beyond 20 μm. The deviation results from errors in DDA which occur for small particles at long wavelengths: The used DDA code applies the so-called “Lattice Dispersion Relation” for assigning dipole polarizabilities [9]. This is based on an analysis of wave propagation on an infinite lattice of polarizable points and, therefore, errors occur for small particles as can be seen here at long wavelength.

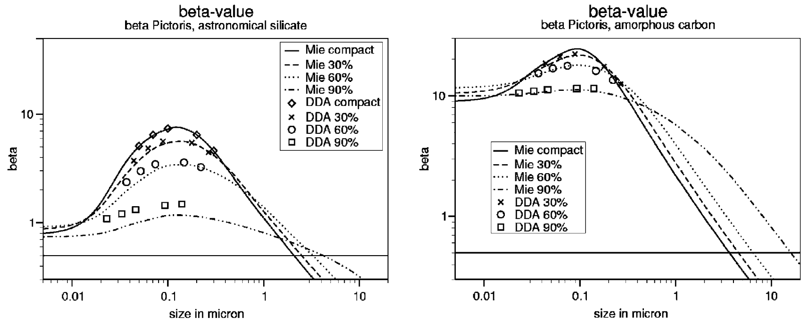

Fig. 3. β-value of β Pictoris for compact and porous spherical particles of astronomical silicate (left) and amorphous carbon (right).

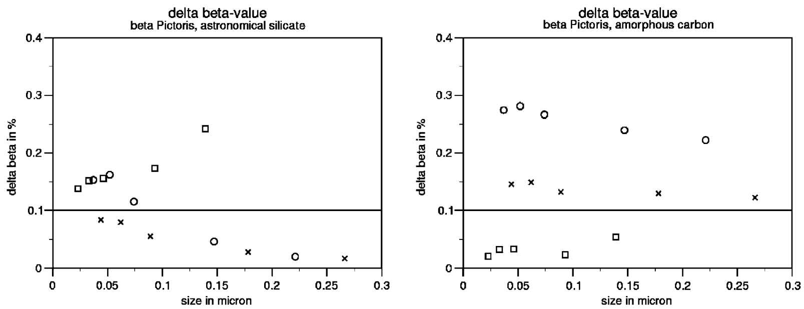

Fig. 4. Differences between β-value determines with Mie and DDA of β Pictoris for compact and porous spherical particles of astronomical silicate (left) and amorphous carbon (right), symbols are same as in Fig. 3.

The β-values for dust grains in the system of β Pictoris calculated for compact and porous spherical particles consisting of astronomical silicate (left) and amorphous carbon (right) are shown in Fig. 3. Calculations with Mie theory depicted as lines are compared with DDA calculations shown as symbols for different particle sizes and porosities. For bigger grains the number of dipoles is very large and calculations exceed our computer capabilities. The dynamical cut-off limits indicated by the horizontal lines ($\beta = 0.5$) in the figures will be discussed below.

We first discuss the β-values. For both theories compact spherical grains and grains of 30%, 60%, and 90% porosities are calculated and shown in the plots. The β-values calculated with Mie theory and DDA are identical for compact spherical particles for particle sizes of 0.05, 0.07, and 0.1 μm. Fig. 4 shows the differences between the calculations for DDA and Mie theory for the porous particles of astronomical silicate (left) and amorphous carbon (right). In the calculated size range the differences between Mie and DDA calculations are smaller than 30% for all porosities and sizes considered. We expect that the slopes of the curves are also comparable for larger particles. Given the large variation of β-values with the stellar spectrum as well as with the material compositions the calculations provide an acceptable estimate.

Figs. 5 and 6 present Mie calculations of the β-values for compact and porous spherical particles consisting of astronomical silicate and amorphous carbon. We calculate the β-value for compact spherical grains and for grains with a porosity of 30%, 60%, and 90% in the systems of Fomalhaut

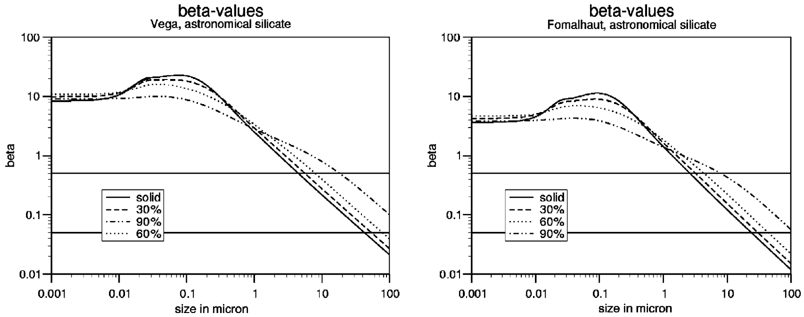

Fig. 5. Calculated β-value of astronomical silicate of solid particles and grains with porosities of 30%, 60%, and 90% for Vega (left) and Fomalhaut (right).

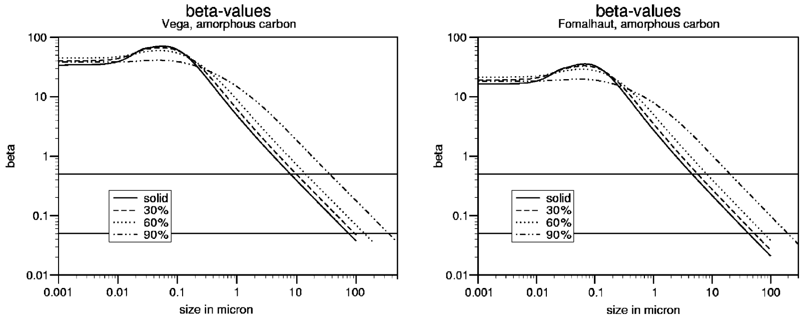

Fig. 6. Calculated β-value of amorphous carbon of solid particles and grains with porosities of 30%, 60%, and 90% for Vega (left) and Fomalhaut (right).

(Figs. 5 and 6(left)) and Vega (Figs. 5 and 6(right)). Compact spheres of amorphous carbon with a size of 0.1 μm reach the highest β-value of 90 in the Vega system while these particles in the Fomalhaut system have values of 35 and in the β Pictoris system of 25 (see Fig. 3(right)). For comparison these particles reach a β-value of only 3 in the solar system. These big variations arise from differences in the spectral energy distributions of the stars which are determined by the size of the star and the photospheric temperature. Vega is a bright A0V main sequence star with an effective temperature of $T_{\rm eff} = 9553 \pm 111$ K [10] while Fomalhaut is a A3V main-sequence star and has a temperature of $T_{\rm eff} = 8800$ K [1]. While β Pictoris is a young main sequence star of spectral type A5 IV and the stellar photospheric temperature is $T_{\rm eff} = 8250$ K [11], the Sun has a temperature of only $T_{\rm eff} = 5785$ K.

As opposed to larger bodies that are mainly influenced by stellar gravity, the dynamics of smaller particles is influenced by the radiation pressure force. This changes the orbits of particles that are released from a larger parent body—either in the case of collisional fragmentation or if the particles are ejected from cometesimals—and are affected by the radiation pressure. The condition for fragments that are emitted in perihelion to stay in bound orbits is

$$\beta < \frac{1}{2}(1 - e'), \tag{6}$$

where e' is the eccentricity of the orbit of the parent body [2]. The line at the β-value of 0.5 indicates this condition of dust ejection for particles in cicumstellar circular orbit ($e'=0$). Particles with higher β-values will be blown out of the system by radiation pressure when they are released by a parent body. Particles emitted by a parent body in highly elliptic orbit with $e' = 0.9$ are ejected for $\beta > 0.05$ (also shown in the figures).

The β-values mainly depend on the stellar temperature and so does the cut-off limit: The higher the temperature of the star the bigger are the particles which can stay in bound orbits about the star. The size limit of compact spherical particles that are ejected by radiation pressure varies for the different stars within a range of 1/10 of micrometers to several 10 micrometers, with small variations depending on the dust material composition and structure. Earlier calculations give the β-values for β Pictoris of compact spherical particles of different materials: NH_3 ice, obsidian, basalt, astronomical silicate, H_2O ice mixed with different fractions of graphite, pure graphite [12], amorphous carbon, and crystalline olivine [2]. The cut-off limits lie in a range of particle sizes between 1.1 and 10 μm.

Considering astronomical silicate particles released from parent bodies in circular orbits we find cut-off limits between 2.5 and 8 μm for Fomalhaut (Fig. 5(right)), between 4.2 and 20 μm for Vega (Fig. 5(left)), and between 2 and 4 μm for β Pictoris (Fig. 3(left)). In the case of amorphous carbon we find cut-off limits between 4 and 30 μm for Fomalhaut (Fig. 6(right)), between 7 and 40 μm for Vega (Fig. 6(left)), and between 4 and 20 μm for β Pictoris (Fig. 3(right)). For large porosities the cut-off limit shifts to larger particles.

4. Summary

We compare Mie calculations of the radiation pressure force to DDA calculations for compact and porous spherical particles and conclude that Mie theory is applicable for spheres with porosities up to 90% for the calculations of the radiation pressure forces. This allows a comparison of the dynamical cut-off limits in the circumstellar systems. DDA calculations describe the scattering and absorption properties of particles of arbitrary shape and also high porosities but need too much computer time and memory to calculate big grains. Calculations to determine the size limit of particles that are ejected from circumstellar systems by radiation pressure are carried out using Mie theory for compact spheres and grains with porosities up to 90%. The cut-off limits lie within a range of 1/10 of micrometers to several 10 micrometers, with variations depending on the dust material composition and the grain structure. The calculated β-values serve as examples for the behavior of dust particles in systems with similar stellar temperatures. We here consider only spherical porous particles. The real grains in the systems may have a different structure such as aggregates if grains are similar to cometary dust particles. For this case our calculations provide a first estimate to be verified with future model calculations.

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