PHYSICS OF DEBRIS DISKS

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ABSTRACT

In the last decade, hundreds of debris disks around main-sequence stars have been detected through infrared excesses in the stellar spectra and some imaged directly. This review outlines essential forces and effects exerted on dust in debris disks and then summarizes key properties of the disks, as determined by these physics. Size and spatial distribution of dust, long- and short-term evolution of disks, as well as interrelations between dust, its parent bodies, and embedded planets are addressed.

1. OBJECTS

Debris disks are optically thin, gas-poor dust disks around main-sequence (MS) stars. Their origin and status can easily be understood in the framework of the conventional planetary system formation scenario. Planetesimals, planetary embryos, and then planets form in a primordial protoplanetary disk around a star. Planetesimals that were neither used to make up planets nor ejected survive this relatively rapid process (~ 10 Myr). These leftovers begin to produce dust by mutual collisions and, possibly, comet-type activity [1, 2], creating a tenuous debris disk. Being continuously replenished by small bodies, the disk can then persist over much of the star's lifetime.

Debris disks must be clearly distinguished from protoplanetary disks — denser disks with a high gas content around young T Tau and Herbig Ae/Be stars. These are disks, in which the planet formation process has probably begun but is not yet completed. Whether or not this interpretation is true, there is one objective criterion to distinguish between protoplanetary and debris disks [3]: presence or absence of gas. Protoplanetary disks still contain gas in considerable amounts. Therefore, the physics of the dust component in protoplanetary disks are gas-dominated. These disks are *not* considered here.

Although only a handful of debris disks have been resolved so far at different spectral ranges from visual to sub-mm, many more images are expected from ongoing ground- and space-based searches. Resolved examples are the disks of Vega [4–9], ϵ Eri [10, 11], α PsA [12], HD32297 [13, 14]. HD107146 [15, 16], η Crv [17], and τ Cet [18]. Two other well-known examples — β Pic [19–22] and AU Mic [23–25], both probably less than 20 Myr old [26, 27], may be too young to be classified as "typical" debris disks and should be treated as "transitional" disks instead. Yet more questionable is "debrisness" of disks around HR 4796A — see [28–31] vs. [32] and HD141569 [33–35]. Surveys have revealed IR excesses in spectra of more than 300 MS stars and show that $\geq 15\%$ of MS stars may possess debris disks [36].

Does the Sun have a debris disk? It is thought that the interplanetary dust cloud, or zodiacal cloud, a tenuous sheet of dust observed up to the orbits of giant planets, is rather a relatively dust-poor inner part in the actual debris disk which encompasses the Edgeworth-Kuiper belt (EKB) region. The extensions, mass, and cross section area of the latter should by far supersede those of the zodiacal cloud. Ironically, this disk has not been observed yet, due to its extremely low optical depth, typical of Gyr-old systems such as the solar system. Nevertheless, there are some indications for its existence in spacecraft data [37].

Interpretation of the rapidly growing bulk of observational data necessitates a major theoretical effort to understand the physics and evolution of debris disks. This paper outlines main physical mechanisms acting in debris disks and essential properties they should determine, touching upon observational aspects only briefly. Detailed overview of the observational results can be found in other recent reviews [e.g. 3, 38, 39].

2. PHYSICAL PROCESSES

2.1. Stellar gravity

The main force keeping the solids in closed orbits is the central star's gravity:

$$\mathbf{F}_g = -G\frac{Mm}{r^2}\mathbf{r}^0, \tag{1}$$

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where G is the gravitational constant, M is the stellar mass, m is the mass of the dust particle, r its astronocentric distance, and $\mathbf{r}^0$ is the a unit vector in the radial direction.

2.2. Radiation pressure

The net force experienced by a grain in a radiation field of the star is

$$\mathbf{F}_{rp} = \frac{GM\beta m}{r^2}\left[\left(1-\frac{\mathbf{v}\mathbf{r}^0}{c}\right)\mathbf{r}^0-\frac{\mathbf{v}}{c}\right], \quad (2)$$

where c is the speed of light, $\mathbf{v} = \dot{\mathbf{r}}$ is the velocity of the dust grain, and β, the radiation pressure to gravity ratio, is a constant depending on the luminosity and mass of the star as well as the grain size, density and optical properties [40].

The radiation pressure force can be split into two components, which are usually treated separately. The velocity-independent constant radial part of (2) is called *direct radiation pressure* force, $\mathbf{F}_{rad}$. A combined effect of the stellar gravity and direct radiation pressure is a ”photogravitational” force,

$$\mathbf{F}_{pg} = \mathbf{F}_g + \mathbf{F}_{rad} = -\frac{GM(1-\beta)m}{r^2}\mathbf{r}^0, \quad (3)$$

that leads to a Keplerian motion around the “effective star” with a reduced mass. For sufficiently small grains, it reduces to zero and for yet smaller grains becomes negative. If a grain is released by a parent body moving in a circular orbit, or originates from a collision of two grains moving in near-circular orbits, a reduction of the stellar mass by half ($\beta = 0.5$) leads already to an unbound parabolic trajectory. Thus small grains are blown away from the star in parabolic or hyperbolic paths; they are called β-meteoroids. Bigger grains that can stay in bound orbits around the star are referred to as α-meteoroids.

2.3. Poynting-Robertson effect

The velocity-dependent terms in (2) determine the *Poynting-Robertson (P-R) drag* force $\mathbf{F}_{PR}$[1]. Being a dissipative force, it causes a particle to lose gradually its orbital energy and angular momentum. Thus all α-meteoroids move in Keplerian ellipses with reducing a and e [41]. On timescales of thousands of years or more the trajectory shrinks to the star.

Recently, Breiter and Jackson [42] found a more accurate solution to the perturbations of orbital elements by the P-R force and showed that very close to the central star, the P-R effect would lead to an eccentricity growth and other consequences not realized before. However, such a behavior is only possible at several radii for the star and is not of importance for the dynamics of debris disks.

[1] To avoid confusion, note that some authors define the P-R force differently, either as the radial part of $\mathbf{F}_{rad}$ or as the whole term with $\mathbf{r}^0$ in (2) [40].

2.4. Collisions

Collisions always play a dual role. On the one hand, they destroy the material: regardless of the outcome of a collision between two particles, both colliders are removed from the system and replaced either with a larger particle (agglomeration) or smaller ones (destruction). On the other hand, this means that collisions generate new particles and therefore act as “sources” of material.

How often do grains collide? The collisional rate increases with the number density of grains, and is proportional to the cross section for the collision, $\sigma = \pi(s_t + s_p)^2$, where s_t and s_p are the radii of target and projectile, respectively. As larger particles are usually present at lower number densities, both factors go in opposite directions. Whether smaller or larger particles are affected most by the collisions, depends on the size distribution. In addition, the collisional rate is proportional to the relative or impact velocity v_{imp} , which is higher for systems with larger typical orbital eccentricities and/or inclinations.

What happens when two grains collide? In contrast to protoplanetary disks, relative velocities in optically thin, gas-poor debris disks are high ($\gtrsim 1\,\mathrm{km\,s^{-1}}$). The minimum velocity needed to disrupt two equal-sized colliders is $v_{min} \approx \sqrt{8Q_D^*}$, where the critical energy for fragmentation $Q_D^* \sim 10^7\,\mathrm{erg\,s^{-1}}$ at dust sizes, yielding $v_{min} \sim 0.1\,\mathrm{km\,s^{-1}}$ (e.g. [43]). Therefore collisions typically destroy or erode the particles and create smaller fragments.

2.5. Stellar wind forces

Instead of a subkeplerian gas disk as in protoplanetary systems, in debris disks one can expect quite a different gas component — stellar wind. Like stellar electromagnetic radiation gives rise to radiation pressure forces, stellar particulate radiation — stellar wind — causes stellar wind forces. Similarly to the net radiation pressure force, the total stellar wind force can be decomposed to *direct stellar wind pressure* and *stellar wind drag*. The momentum and energy flux carried by the stellar wind is by several orders of magnitude smaller than that carried by stellar photons, so that the direct stellar wind pressure is negligibly small. However, the stellar wind drag forces cannot be ignored, because v_{sw}, to replace c in Eq. (2), is much smaller than c.

Consider now the stellar wind drag, which is a corpuscular analog of the P-R drag. The stellar wind drag force, in turn, consists of two parts. The first of them, the *direct stellar wind drag* $\mathbf{F}_p$, is caused by momentum transfer from impacting stellar wind particles. Dynamically, the action of the direct stellar wind drag is similar to that of the P-R drag and the strength of the former amounts, in case of the Sun, typically to ~ 30 to 60% of the latter. Both drag forces are usually treated together by introducing the “effective” (P-R + stellar wind) drag. Another part of the stellar wind drag force, the *indirect or Coulomb stellar wind drag* $\mathbf{F}_c$, constitutes a dynamical friction due to distant encounters between the stellar wind particles and the dust grain. The dynamic effects of the Coulomb drag are the same as those of the direct drag, but its strength — at least for the Sun — is by about three orders of magnitude less.

2.6. Interactions with planets

Gravitational perturbations by planets, if there are any in the disk, have direct consequences for both global and local distributions of dust. We classify interactions between the dust and the planet into three cases: (distant) non-resonant perturbations, resonant perturbations, and close encounters.

Non-resonant perturbations take place when, first, the semimajor axes and eccentricities are such that encounters between a dust grain and the planet are not possible, and second, their orbital periods are not in a rational commensurability. Thus non-resonant gravitational perturbations are the most common effect one expects in the dust cloud, since it is not restricted to a certain set of initial orbital elements. This kind of interaction causes periodic oscillations of positional orbital elements of the grain, as well as precession of lines of apsides and nodes, making the dust complex rotationally-symmetric.

Resonant perturbations take place when the mean motions (or other typical dynamical frequencies) come into a rational commensurability. A grain usually reaches commensurability when drifting toward the star due to the P-R force. One can speak of primary (1:1), external, and internal mean-motion resonances (MMRs), depending on whether the orbital period of the grain is equal to, greater, or less than that of the planet. Typical features of MMRs can be summarized as follows [44]. While a grain is trapped in a primary MMR, the eccentricity and inclination both decrease, and the grain is moving in a stable tadpole orbit around L_4 or L_5. As time elapses, the P-R and stellar wind drag gradually transform tadpole orbits into horseshoe orbits, enabling a particle to approach the planet. Repeated close encounters eventually force the grain to quit the resonance. When the grain is trapped in an external MMR, eccentricity grows up to a certain value, called "universal eccentricity" [45]. Again, the presence of dissipative forces — P-R and stellar wind drag — makes the resonance trapping only temporary, and the particle is destined to eventually leave the stable resonant orbit. Finally, trapping in interior MMRs is generally less efficient than in exterior ones [46]. Still, temporary trapping is possible. While trapped in such a resonance, the grain decreases its eccentricity, and — though very slowly — the inclination decreases as well. The above picture applies to the so-called e-type MMRs, which are the most typical for grains in prograde orbits about the star. In some cases, especially for retrograde motions, the I-type resonances are possible, accompanied by a considerable pumping of inclinations [44]. In addition to MMRs, another broad class of resonances, the so-called secular ones, exists. Secular resonances appear when there is a commensurability between the precession rates of apsides and/or node of grain's orbits with those of a planet. These resonances may be important, for example, for delivery of asteroids to the inner solar system [47], fine sculpturing of the EKB [48], and even for cometary orbits in the β Pic system [49]. It may also be important for the dynamics of the asteroidal dust grains drifting from the asteroidal belt toward the Sun [50].

In some cases, the grain may be in a planetary-crossing orbit, making *close encounters* possible. This happens, for example, when a grain is ejected into a planet-crossing orbit from a parent body, Close encounters are also common at the "resonance-quitting phase" described above. Alternatively, the grain may spiral toward the Sun due to the P-R and solar wind drag forces and approach a planetary orbit, without being captured in resonance. Close encounters lead to unstable motion, often referred to as "gravitational scattering" of the orbital elements. Eventually, the grain either hits the planet or is ejected out of the system.

2.7. Sublimation

Sublimation, or transition of dust grains from the solid to the vapor state, occurs in the vicinity of the star. The radius of the sublimation zone depends strongly on the material and porosity of the dust grains and on the luminosity of the primary. In the solar system, silicate and carbonaceous grains sublimate at 2–4 solar radii from the Sun [e.g. 51, 52], but for luminous stars and other materials (extreme example: ices) sublimation zones may be very large.

2.8. Lorentz force

Dust grains interacting with the stellar radiation and stellar wind acquire electrostatic charges due to a variety of effects (photoelectron emission, secondary electron emission, sticking of electrons and ions etc.). Under the presence of the star's magnetic field, they experience the Lorentz force. Its influence on the dynamics of interplanetary dust particles in the solar system is well understood. As the dust grains move through the sectored magnetic field of the Sun with alternative polarities, the Lorentz force rapidly changes its direction. The mutual near-cancellation of these contributions is not complete, however: on long time spans the Lorentz force results in a "Lorentz diffusion", i.e. essentially stochastic changes in a, e and, most importantly, inclination i [53–56]. The effect becomes significant for grains less than several micrometers in size. For yet smaller, submicrometer particles, the Lorentz force becomes very strong and provides an additional mechanism for ejection of particles out of the solar system [57].When modeling debris disks around other stars, the Lorentz force is usually ignored. One reason for that is that it affects relatively small grains, which make only a minor contribution both into the mass budget and the total cross section area of the dust disk. Still, the Lorentz force might be of importance for the dynamics of β-meteoroids.

2.9. Other forces and effects

A number of more exotic effects exists that may influence the dust particles themselves and their dynamics [40, 58]. Some are related to the fact that the star is not a point-like object (differential Doppler effect, radiation pressure from an extended source). Others stem from rotation of grains (windmill effect, Radzievsky effect). Still others are related to modification of the grain properties by the environment (sputtering by plasma or interstellar grains, packing effect, electrostatic breakup), etc. None of these, however, seem to be of generic importance for debris disks, requiring either closeness to the star, smallness of particles, or special material compositions.

3. UNPERTURBED DEBRIS DISKS

3.1. General scenario of dust evolution

We now attempt to synthesize knowledge of numerous forces and effects affecting the dust grains into a consistent picture of a "typical" debris disk as a physical system. In this section, we address the unperturbed case — a planet-free disk.

A general scenario of the dust evolution can be summarized as follows. Parent bodies steadily supply the disk with solids. Initially, most of the mass is released in the form of larger meteoroids, which then undergo collisional grinding. Subsequent evolution of dust-sized particles ($< 1\,\mathrm{mm}$) is largely controlled by three players: stellar gravity, radiation pressure forces, and mutual collisions. Their relative importance depends primarily on the disk's optical depth τ which determines collisional rates.

In systems with roughly $\tau < 10^{-5}$, exemplified by the solar system's debris disk, mutual collisions limit the lifetime of grains that are larger than $\approx 100\,\mu\mathrm{m}$ [59]. Grains smaller than that, yet large enough to stay in bound orbits, are subject to the P-R force. They migrate toward the star where they sublimate. The tiniest collisional debris, for which β-ratio is large enough, are removed from the disk by direct radiation pressure. If $\tau > 10^{-5}$, which is the case for all extrasolar debris disks resolved up to now, the scenario is different. P-R drag is inefficient [19, 60, 61], as the collisional lifetimes are much shorter than the P-R times. The "stone mill" of mutual collisions permanently destroys larger particles and generates smaller fragments. Removal of fine debris by stellar radiation pressure is the main loss channel of material in such systems. In either scenario, all loss mechanisms listed here imply grain lifetimes of < 1 Myr, showing that debris disks cannot be primordial.

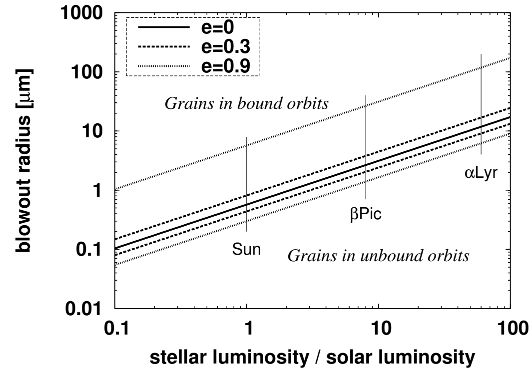

Figure 1. Grain radius that separates particles in bound and hyperbolic orbits, as function of star's luminosity (assuming dust bulk density of $2\,\mathrm{g/cm^3}$, a unit radiation pressure efficiency, and a standard mass-luminosity relation for MS stars). Different linestyles are for different typical eccentricities of parent bodies. Grains between two lines of the same style may be in both types of orbits. *From Krivov et al. [62]* (reproduced by permission of ESO).

3.2. Size distribution

At any time, a disk contains two distinct populations of dust: bigger grains in bound orbits around the star (α-meteoroids) and smaller ones that are placed by radiation pressure in hyperbolic orbits, but are steadily replenished through collisional cascade (β-meteoroids). A boundary between the two populations lies usually at 0.1–$10\,\mu\mathrm{m}$, depending mainly on the mass and luminosity of the star and optical properties of grains, as well as on typical eccentricities of the dust parent bodies (Fig. 1).

A typical size distribution of dust obtained by modeling of the α Lyr disk [62] is depicted in Fig. 2. Most noticeable is the peak at the particle size connected with $\beta \lesssim 1$, where the radiation pressure is almost as strong as gravity (cf. Fig. 1). Below this size bound orbits are impossible, and the grains are blown away at a disk-crossing timescale of the order of 10^2 to 10^3 yr. Due to the lack of possible impactors, grains slightly above this size limit are overabundant, thereby reducing the number of grains of the next larger population, and so on. This dependence produces a well-known wavy pattern in the size or mass distribution [e.g., 63, 64, 65], whose wavelength depends on the ratio of the average impact energy available and the impact energy needed to disrupt a given target. We note that a realistic disk cannot be built up of a perfectly homogeneous material, which implies a dispersion of densities, fragmentation energies etc. This could weaken or smear the waviness of the size distribution. For disks around stars with lower luminosities than Vega's, the size distribution pattern seen in Fig. 2 shifts to the left, as the blowout limit falls to smaller sizes (Fig. 1).

Although the total cross section is dominated by particles with radii somewhat above the blowout limit, the pres-

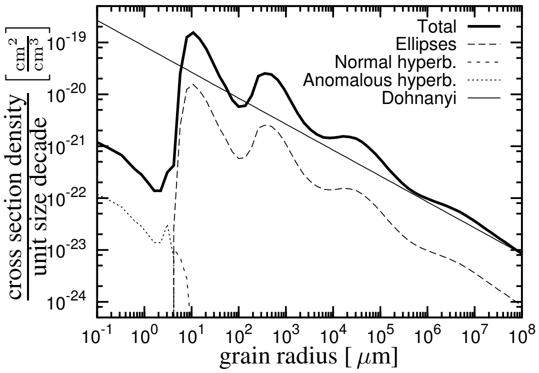

Figure 2. Size distribution in the Vega disk (model). Shown are the net distribution (solid line) and relative contributions made by grains in bound elliptic orbits (dotted line), hyperbolic orbits, and "anomalous" hyperbolas open outward from the star (dashed lines), Anomalous hyperbolas are orbits of smallest grains with $\beta > 1$. Dotted and dashed lines are shifted down by one order of magnitude for better visibility. Straight line represents the classical Dohnanyi distribution $n \propto s^{-3.5}$. *From: Krivov et al. [62]* (reproduced by permission of ESO).

ence of small grains in hyperbolic orbits is important as well, e.g. for the disk colors. The entire two-population model provides a good fit to both spectrophotometric and polarimetric data [66].

3.3. Spatial distribution

The radial profile of the disk can be characterized, for instance, by the number density $n(r)$, surface density $\sigma(r)$, normal optical depth $\tau(r)$, or surface brightness $B(r)$ as functions of distance from the star r. All these are usually approximated by power laws with different indices, although the index may change from the inner to the outer part of the disk. Obviously, σ and τ have the the same index, which we denote α: $\sigma \propto \tau \propto r^{-\alpha}$. For a non-flared disk, whose vertical thickness is proportional to r, the corresponding $n \propto r^{-\alpha-1}$. The distribution of B depends, however, on the temperature of the star, wavelength of the observations, as well as the sizes and temperatures of the dust grains. For example, let the disk be composed mainly of grains larger than the peak wavelength in the stellar spectrum, and assume that the disk is observed at wavelengths much larger than the peak of the thermal emission of grains. In this case, the temperature is proportional to $r^{-1/2}$, the emitted black-body energy is proportional to the temperature (Rayleigh-Jeans regime), and $B \propto r^{-\alpha-1/2}$. At shorter wavelengths, the brightness profile can be as steep as $B \propto r^{-\alpha-5/2}$.

What determines the index α? Of course, the radial distribution of directly invisible parent bodies matters. Unfortunately, it is fairly unknown for all extrasolar debris disks. By analogy with the solar system, one may expect the parent bodies to concentrate in certain radial zones. Moreover, for "dynamically full" systems like our own solar system, it is likely that the parent bodies occupy relatively narrow annuli around the star, "planetesimal belts". Would that mean that the dust material released from them should show up in form of pronounced peaks at the location of the belts? Not necessarily. As, for instance, Wyatt [61] has shown, the answer depends on whether the (cross section-dominating) particles essentially stay at the same distances before they are lost, or are able to considerably migrate before that.

Several analytic solutions for the radial profile are known. A collisionless disk without sources and sinks and with grains in circular orbits, spiraling in circular orbits toward the star due to the P-R force, has $\alpha = 0$ [e.g. 67]. The same system with eliminating collisions (assuming like-sized grains) is characterized by $\alpha = 1/2$ [e.g. 69, 70]. Another simple-structured debris disk would be a cloud of grains moving outward from the center in hyperbolic trajectories. This solution, which applies to β-meteoroids in the interplanetary space, is described by $\alpha = 1$ [e.g. 71, 68]. Interestingly, the overall distribution of dust in the solar system can be fitted with $\alpha \approx 0.3$ [59].

A closer look at the problem uncovers another mechanism that influences the radial slope, even if the transport mechanisms (P-R drag) are absent. Krivov et al. [62] considered a narrow planetesimal belt producing relatively large dust grains, which then undergo a standard collisional grinding, producing smaller and smaller fragments. These debris will have progressively larger β-ratios. As the result, they will be sent by radiation pressure to elliptic orbits with pericenters at their birthplace, but apocenters much farther out from the star. Of course, more distant apocenters correspond to smaller grains, which make a lesser contribution to the optical depth and brightness. Nevertheless, modeling shows that the fragments created at the location of the planetesimal belt affect the optical depth out to a considerable distance, setting an upper limit to the radial slope α; for the Vega disk, between 1 and 2. Even a ring-like or toroidal parent population with a sharp outer cutoff would eventually lead to a slope not steeper than this, after the disk has reached a steady state. The relaxation timescale varies from one system to another. For Vega, it takes $\sim 10^5$ years [62].

3.4. Long-term decay of disks

A population of planetesimals, acting as a source of dust for the debris disk, is a collisionally-evolving system without replenishment. As a result of catastrophic and cratering collisions the planetesimal disk is gradually depleted, usually on Gyr timescales. Accordingly, the masses and optical depths of debris disks should decay with time. Recent statistical surveys [see 39, and references therein] support this expectation: τ reduces from $\sim 10^{-2}$ for Myr-old stars to $\sim 10^{-7}$ for Gyr-old ones. Analytic models for the collisional depletion of the parent body populations and dust production in debris disks show that the dustiness of the evolved disks should go down approximately as a reciprocal of time [72, 62].

3.5. Non-stationarity

The properties discussed above are expected for a debris disk in a steady state, resulting from the dynamical equilibrium between sources and sinks of the constituent material. However, substantial fluctuations in the disk parameters can be expected, most notably from major collisions between large planetesimals in the disk that produce copious amounts of dust at a time. Furthermore, dust debris released in such events can trigger the so-called dust avalanches, provided that the disk is dusty enough [73]. The dust cloud gradually spreads and then disappears in the "smooth" component of the disk. Major collisions can especially be important at earlier stages of the disk evolution, such as the heavy bombardment era in the solar system, but even in the present-day solar system they may be substantially contributing to the interplanetary dust cloud [e.g. 74] and the EKB dust disk [75, 76, 77]. For the parameters of a minimum solar nebula, and assuming a distance of 100 AU from the star, a collision of two 1000 km-sized objects is required to produce an observable dust cloud. Such clouds would form once per several Myr and remain distinct for roughly 10^4 to 10^5 years before they are broadened by the Keplerian shear into a ring [78].

4. DEBRIS DISKS PERTURBED BY PLANETS

4.1. Non-resonant structures

Non-resonant planetary perturbations are known to be responsible for the rotational symmetry of the whole zodiacal cloud in the solar system and a small tilt of its symmetry plane off the ecliptic [e.g. 50]. Jupiter usually produces the strongest perturbations, but also Venus perturbations were shown to be important for shape and orientation of the inner zodiacal cloud [79].

Non-resonant perturbations were studied for other debris disks with presumed planets as well. If the planetary orbit is slightly eccentric, apsidal lines of dust particle orbits tend to align with that of the planet's orbit. The azimuthal distribution of dust would reveal an offset then. Observed asymmetry of the wings in the disks of β Pic, for instance, is attributed to this effect [80]. Similarly, if the planetary orbit is slightly inclined to the symmetry plane of the disk, a warp is induced in the disk. This effect should be observable for disks viewed nearly edge-on, and provides a natural explanation to the warps seen in the disks of β Pic [80] and AU Mic [24]. Mouillet et al. [80] have shown that the warp propagates outward from the star with time, so that its position can even shed light to the system's age.

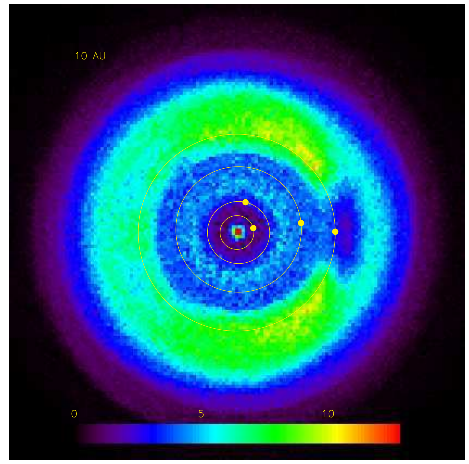

Figure 3. Spatial density of EKB grains with $a \approx 10\,\mu$m (model). Neptune clears up the space around its orbital position and creates leading and trailing resonant "blobs" of dust. Saturn and Jupiter act as barriers, preventing EKB dust from penetrating the inner solar system. *From: Liou and Zook [86]* (reproduced by permission of the AAS).

4.2. Resonant structures

Resonant perturbations were investigated in an attempt to describe the transport of cometary and asteroidal dust to the inner solar system [81, 82]. According to [82], about 20% of all cometary grains arriving at 1AU were previously trapped in MMRs with Jupiter, which decreases eccentricities and inclinations of cometary grains. Vice versa, the passage of the asteroidal particles through secular resonances at the inner edge of the asteroid belt increases eccentricities and inclinations of the asteroidal grains [50]. Both effects would make the orbital distributions of cometary and asteroidal particles at 1 AU hardly distinguishable. Liou and Zook [83] found that a significant fraction of micron-sized grains released from the asteroidal belt are thrown into the 1:1 MMR with Jupiter and may give rise to "Trojan dust clouds" of Jupiter. While these are not yet discovered observationally, the dust ring around the Earth orbit that owes its origin to the same resonant mechanism, is successfully observed [84, 85].

In the outer solar system, Liou and Zook [77, 86] considered perturbations induced by jovian planets on the EKB dust disk. They found (Fig. 3) an efficient trapping of dust by Neptune in MMRs (which produces arcs of dust co-orbital with the planet) and efficient ejection of dust outside the solar system by Jupiter and Saturn. Were the solar system observed from outside, the presence of at least Neptune and Jupiter could be obvious merely from the analysis of the images of the dust disk.

Similar research has been done for ϵ Eri [87, 88], α PsA [12], and other systems. A systematic overview of resonant structures that a single planet can form in a debris disk was painted by Kuchner and Holman [89].

Substructure in the disks is, however, not necessarily produced by presumed planets and may also reflect their intrinsic clumpiness (recent collisions between the large planetesimals). Another problem is that catastrophic grain-grain collisions may smear out the planet-induced structure already at moderate optical depths $\tau \sim 10^{-5}$ to 10^{-4} [90, 91, 92]. Yet another problem may occur with the P-R effect as a delivery mechanism of dust into resonant locations. It is well known [see, e.g. 86, 93] that capture in MMRs is most efficient for larger particles with small β, for which the P-R decay is the slowest. This also means, however, that the P-R timescales for such grains may be too long compared to the collisional timescales — at least for optical depths of $\tau \gtrsim 10^{-4}$ or so, which is the case for all debris disks directly imaged so far. Below we discuss a possible alternative scenario that does not require the P-R effect.

Nevertheless, future observational facilities such as ALMA should be able to image tenuous debris disks at a much lower, solar system-like, optical depth level. For those systems the standard mechanism outlined above must work. Therefore, observations of the fine structure together with dynamical simulations may help to pinpoint unseen planets, even with a moderate mass. These issues are addressed in detail by Moro-Martín (this volume).

4.3. Inner gaps

We now consider formation of inner gaps in the disk by an alleged planet. The importance of these studies stems from the fact that the presence of inner gaps can be inferred reliably from the spectral energy distribution even for non-resolved disks, therefore providing a tool for indirect planet searches [94].

The discovery of the inner depletion zone in the β Pic disk with the radius of several tens of AU [e.g., 95] gave rise to the idea that the gap may be caused by the presence of a planet. The proposed mechanism for clearing up the inner region was again a temporary trapping of grains by the planet in outer resonances, which stops the inward motion of dust toward the star [96, 97, 98, 45]. Evaluation of the resonance trapping timescales confirmed that capture has a temporary nature: after typically $\sim 10^5$ to 10^6 revolutions a particle is expelled out of the resonance by close encounters. Most recently, Moro-Martín and Malhotra [99] (see also Moro-Martín, this volume) readdressed the issue and described "dust outflows" by embedded planets as a general phenomenon, and investigated the dependence of its parameters on planet's mass and orbital elements, as well as on the particle size.

In all the studies listed above the P-R effect, possibly enhanced by the stellar wind drag, was assumed as the mechanism that delivers the dust particles from outside, either to outer resonance locations, or to planet-crossing orbits directly. Indeed, in debris disks (as opposed to protoplanetary disks) it is actually the only known mechanism for the inward transport of dust material. Is the P-R effect efficient enough? As noted above, one can doubt that it can be responsible for formation of resonant structures, because only relatively large particles with small β have a high probability to get trapped, whereas the P-R timescales for such grains are too long compared to the collisional timescales. In the solar system, these would be roughly particles $> 10\,\mu$m. However, smaller grains with larger β can indeed be brought by the P-R drag to planet-crossing orbits directly. For the solar system, the relevant sizes would be, crudely, between $0.5\,\mu$m and $10\,\mu$m. Still, some of the particles may pass the planet orbit too quickly, thus avoiding close encounters and getting the chance to drift further inward. On any account, further effort is required to clarify the question of whether the P-R effect plays a crucial role in formation of inner gaps.

4.4. Planets and parent body populations

When modification of the debris disks by embedded planets is discussed, it is usually meant that gravity of planets exert perturbations on the motion of dust particles directly. However, a more general — and more fundamental — effect is that planets control distribution of dust parent bodies in the system. Over the whole planet formation process, growing and possibly migrating planets heavily affect planetesimals remaining in the system and in that way sculpture planetesimal and cometary belts.

Consider, for instance, the density patterns like those observed in the ϵ Eri system. As discussed above, these are usually attributed to trapping of *dust grains* in resonances with a planet. Alternatively, these features may stem from clumps of *planetesimals* which have been captured in resonances with the planet (like Plutinos locked in the 3:2 MMR with Neptune, or Greeks and Trojans in the primary resonance with Jupiter) [91, 92]. A large fraction of dust which these bodies produce may stay locked in the same resonance. The same line of reasoning should apply to inner gaps interior to planet's orbit: the planet, or planets not only act as barriers for dust coming from outside, they simply prevented planetesimals from staying inside, thereby suppressing the dust production in the inner region.

5. CONCLUSION

Circumstellar debris disks discussed in this paper are as typical of young and mature planetary systems as planets themselves and small bodies, leftovers of planet formation. Due to its large total cross section area, dust is much easier to observe than planets, not speaking of planetesimals. On the other hand, as shown in this short review, the distributions of dust reflect distributions of the parent bodies, are sensitive to the presence of embedded perturbers, and bear important "memory" of the planetary formation process in the past. Hence debris disks can be used as indicators of directly invisible small body populations, alleged planets, and evolutionary stages of planetary systems. Deciphering the messages hidden in debris disks is an uneasy task, however. As also shown in this review, many properties of debris disks are driven to large extent by their "internal" physics, which are complex and depend on poorly known optical and mechanical properties of dust. Further work, both theoretical and observational, is necessary to remove ambiguities in the interpretation of debris disk observations and to fully exploit the potential of debris disks as an instrument in planetary system studies. The effort will be rewarding: more insight into origin, evolution, and properties of extrasolar planetary systems.

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