Optimal design and tolerancing of occulters for finding extrasolar planets

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The Spirit of Lyot: Direct Detection of Exoplanets and Circumstellar Disks

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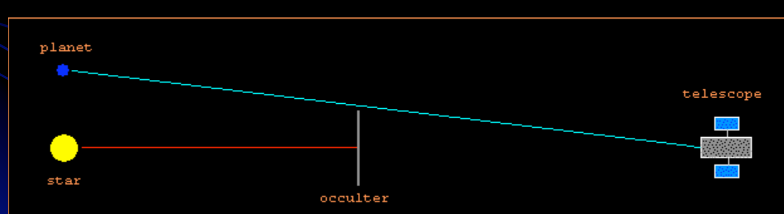
What is an occulter?

We are examining the possibility of finding planets with an *occulter*, a spacecraft that:

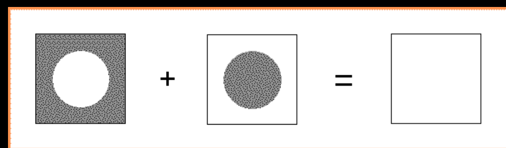
- lies between a telescope and a star
- blocks the starlight, while the planet light goes around the edge
- creates a dark shadow in which the telescope sits



(not to scale)

Calculating the shadow

Babinet's principle states the sum of an electric field due to a hole and another due to a hole-shaped disk add to produce a field as if there had been no obstacle at all.



If the hole is partially transmitting and radially-symmetric, we can write the Fresnel-propagated electric field as:

$$E(\rho, Z) = E_0 e^{\frac{2\pi i Z}{\lambda}} \frac{2\pi}{i\lambda Z} \int_0^R A(r) J_0\left(\frac{2\pi r\rho}{\lambda Z}\right) e^{\frac{\pi i}{\lambda Z}(r^2+\rho^2)} r\,dr$$

and the field from 'no obstacle' as: $E(Z) = E_0 e^{\frac{2\pi i Z}{\lambda}}$

Occulter profiles

The Fresnel diffraction formula for a circularly symmetric occulter with transparency given by A(r) is thus:

$$E(\rho, Z) = E_0 e^{\frac{2\pi i Z}{\lambda}} \left(1 - \frac{2\pi}{i\lambda Z} \int_0^R A(r) J_0\left(\frac{2\pi r\rho}{\lambda Z}\right) e^{\frac{\pi i}{\lambda Z}(r^2+\rho^2)} r\,dr\right)$$

Previous A(r) have included:

Hypergaussian
(Cash 2006)

$$A(r) = \begin{cases} 1 & \text{if } r \le a \\ e^{-\left(\frac{r-a}{b}\right)^n} & \text{if } a < r \le R \\ 0 & \text{if } R < r. \end{cases}$$

Polynomials
(Copi and Starkman 2000)

$$A(r) = \begin{cases} 1 & \text{if } r \le a \\ 1 - \sum_n C_n y^n & \text{if } a < r \le R \\ 0 & \text{if } R < r. \end{cases}$$

$$y = \frac{(r/R)^2 - \epsilon^2}{1-\epsilon^2}$$

Optimization

We propose finding an A(r) by directly optimizing the profile of the transparency.

- Want the occulter to be
  - as close as possible
  - as small as possible
  - viable over the largest bandwidth possible
  - providing the largest shadow possible
- Want the shadow to have intensity less than 10^{-10}
- Choose the occulter with the smallest "area", as this makes the optimization easier to do.

- The optimization problem would then look like this:

$$\begin{array}{lll} \text{Minimize} & \int_0^R A(r) r dr & \\ \text{subject to} & \frac{|E_o(\rho)|^2}{|E_0|^2} \leq 10^{-10}, & 0 \leq \rho \leq \rho_{max} \\ & 0 \leq r \leq R, & 0 \leq A(r) \leq 1 \end{array}$$

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Optimization (II)

This problem is:

- infinite-dimensional
- quadratic program
- at a single wavelength

We can modify it to be:

- a linear program (constrain the real and imaginary part of the E-field)
- finite-dimensional (discretize R and ρ)
- at multiple wavelengths (repeat constraints for different λ)

Additional constraints

Useful to require the profile to get smaller as the radius increases:

$$A'(r) \le 0$$

One concern that has been raised about occulter designs is the sensitivity to error. One way to partially mitigate this is to require the profile to be smoother by bounding the second derivative:

$$-\sigma \le A''(r) \le \sigma$$

To improve robustness, we may also use σ as the constraint in the optimization.

$$\begin{array}{ll} \text{Minimize} & \sigma \\ \text{subject to} & -\sigma \le A''(r) \le \sigma \\ & \vdots \end{array}$$

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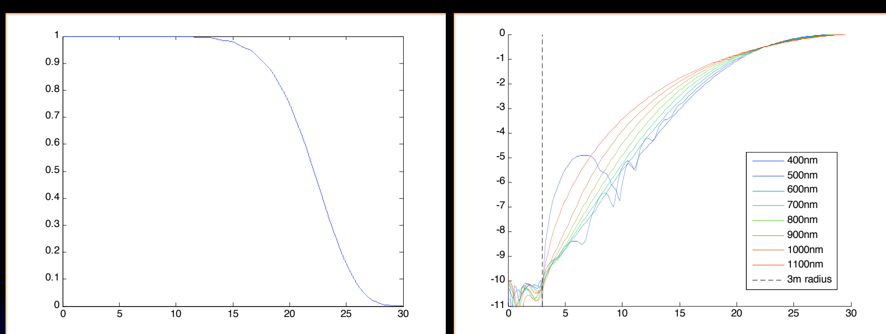
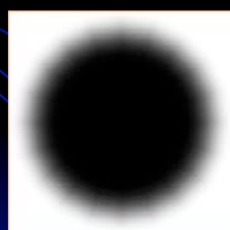
Typical results



Left: Profile of a transmissive occulter located at 100,000 km from telescope. *Right*: Shadow in the band 400nm-1100nm.

Petalization

One way to design an opaque occulter is to approximate a transmissive occulter with an opaque one, using a series of “petals”, which are structures on the edge of the occulter with azimuthal symmetry. (See *Vanderbei et al. 2003*.)



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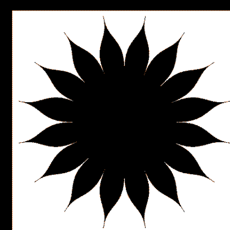

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Petalization (II)

At a given r, the total fraction of the circle at that radius that is opaque is equal to A(r).

The effect of petalizing the opaque occulter is to add a set of perturbation terms:

$$
\begin{aligned}
E_{o,\mathrm{petal}}(\rho,\phi) &= E_{o,\mathrm{apod}}(\rho) \\
&\quad - E_0 e^{\frac{2\pi i z}{\lambda}} \sum_{j=1}^{\infty} \frac{2\pi(-1)^j}{i\lambda z} \left(\int_0^R e^{\frac{\pi i}{\lambda z}(r^2+\rho^2)} J_{jN}\left(\frac{2\pi r\rho}{\lambda z}\right) \frac{\sin\left(j\pi A(r)\right)}{j\pi} r\,dr \right) \\
&\quad \times \left(2\cos\left(jN(\phi-\pi/2)\right)\right)
\end{aligned}
$$

where N is the number of petals (assumed even). The shadow cross-section remains similar close to the axis.

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Optimal petalization

Another solution: instead of using the unperturbed integral as the contrast constraint, use the perturbed integral.

- Optimizes the shape directly
- Not circularly symmetric anymore!
  - Need constraints on each pair of (ρ, φ)
- There will be two specific values of φ that maximize and minimize the perturbation
  - These values can be determined analytically
- Result: twice the number of constraints, but potential for fewer petals

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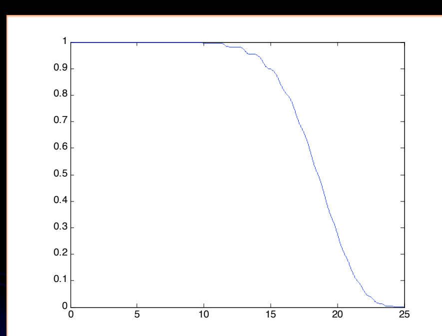
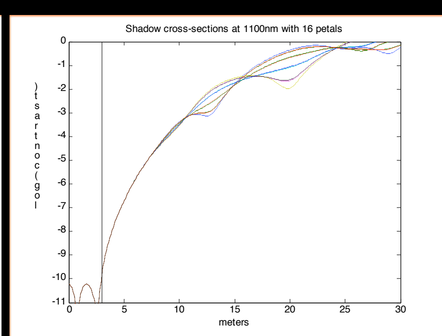
Typical results





Left: This is the shape of a petal as a function of radius, for a 16-petal occulter designed to work in the 0.4um-1.1um range, made with optimal petalization.

Right: Shadow at 1.1um. The different colors are cross-sections at different values of φ.

Error analysis and optimization

As a final note, we can include certain error terms in the optimization directly.

In the Fresnel integral, Z appears with λ as λZ every time it appears. If we note that:

$$(Z + \Delta Z)\left(\lambda - \frac{\lambda \Delta Z}{Z + \Delta Z}\right) = \lambda Z$$

we can extend the bounds of our optimization in λ accordingly to account for this error. This will guarantee that a deviation in Z does not affect the contrast in the deep shadow over the desired spectral band.

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Other areas of analysis

We are looking at effects of other error sources:

- off-axis sources
- translation errors in the occulter or telescope positioning
- rotation errors in the occulter or telescope positioning
- changes in occulter shape
- edge scatter

These would need to be taken into consideration when setting up an optimization—they determine the shadow depth and shadow size.

References

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