Performance study of integrated coronograph-adaptive optics designs

In the Spirit of Bernard Lyot, UC Berkeley 2007

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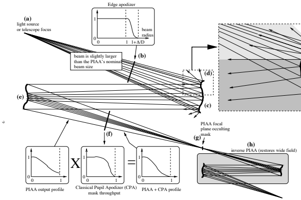
High contrast for exo-planet detection requires extreme performances from both the coronagraph and the adaptive optics

Pluzhnik et Al. (2006)



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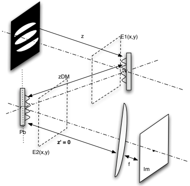
Pueyo, Kasdin (2007), Shaklan et Al. (2006)



High Throughput
Large deformation of the mirrors
On-Sky magnification
BroadBand contrast
No wavefront correction

Designed with ray-optics, modified according to diffractive optics

Low Throughput
Small deformation of the mirrors
No On-Sky magnification
BroadBand contrast
Wavefront correction

Designed with Fresnel diffractive optics

GOAL: Control two sequential deformable mirrors in order to improve the performances of a classical apodiser while doing wavefront control.

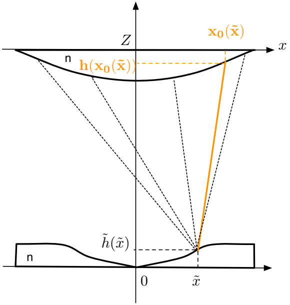
Problem Studied

Starting from the observation that two sequential Deformable Mirrors are a modified version of PIAA we looked into relaxing the contrast constraint of a classical apodiser, and thus increase throughput and decrease IWA, while maintaining a 10^{-10} extinction using the DMs

Our approach is **one dimensional** and follows these three steps:

- We need to understand the physics of the propagation between two curved surfaces and how it limits the contrast of a pure PIAA.
- We need to understand how a classical apodiser breaks down the propagation induced contrast limit
- We need to design control algorithms for the DMs in order to maintain a 10^{-10} contrast in the presence of wavefront aberrations

2 DMs a pupil mapper



For a given $\tilde{x}$, of all the incident wavelets emitted by the surface M_1, the one that minimizes Optical Path Length is coming form the point $x_0(\tilde{x}), h(x_0(\tilde{x}))$. That point is defined by:

$$h'(x_0) = \frac{x_0 - \tilde{x}}{nS(x_0,\tilde{x}) + h(x_0) - \tilde{h}(\tilde{x})} \tag{1}$$

Enforcing the constraint of a flat wavefront coming out of M_2, constant Optical Path Length Q_0, leads to the second differential equation of pupil mapping:

$$\tilde{h}'(\tilde{x}) = \frac{x_0 - \tilde{x}}{nS(x_0,\tilde{x}) + h(x_0) - \tilde{h}(\tilde{x})} \tag{2}$$

Finally, energy conservation yields the relationship between the mapping and the final E-field apodisation $A(\tilde{x})$

$$\boxed{A(\tilde{x})^2 = \frac{dx_0}{d\tilde{x}}} \tag{3}$$

Outgoing field

After some algebra, the Huygens integral simplifies to

$$E_{out}(\tilde{x}) = \frac{e^{i\frac{2\pi}{\lambda}Q_0}}{\sqrt{i\lambda Q_0}} \int e^{i\frac{\pi}{\lambda}\frac{d\tilde{x}}{dx_0}\frac{1+(1-n^2)h'(x_0)^2}{S(x_0,\tilde{x})}(x-x_0(\tilde{x}))^2+o(x-x_0)^3} dx \approx \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{i\frac{\pi D^2}{\lambda A(\tilde{x})Z}(x-\frac{x_0(\tilde{x})}{D})^2} dx \tag{4}$$

Since it is a modified Fresnel Integral, function that is tabulated in most commercial softwares, it provides a method for quick simulations of propagation effects for pupil mapping or 2 sequential DMs

Simulations of the contrast limitation

Limitation with finite optics

In the case of ray optics, that is $D \sim \infty$ or $\lambda \sim 0$, a closed from exists and yields the classical result:

$$\boxed{E_{out}(\tilde{x}) = A(\tilde{x})} \tag{5}$$

Simulation of a 10^{-10} pupil mapping, with $z = 1$ and $\lambda = 600$ nm. The diameter of the optics D varies from 1 cm to 1 m.

(PropPIAAmovie.avi)

Need for hybrid designs

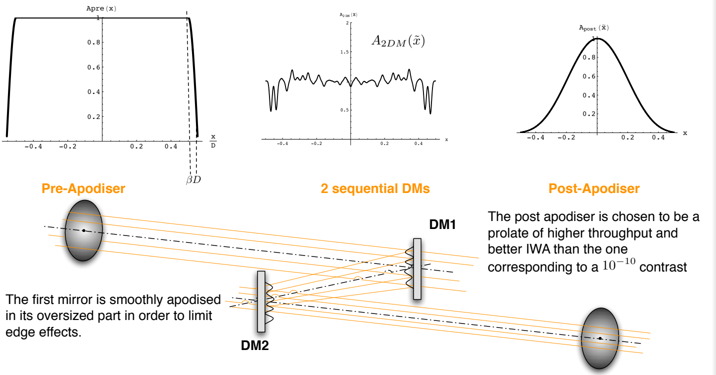
The phase oscillations of $E_{out}(\tilde{x})$ are responsible for the propagation induced contrast degradation. As proposed by Guyon et Al (2006), we mitigate them using a Classical Pupil Apodisation / PIAA hybrid design.

Classical apodiser with two sequential DMs

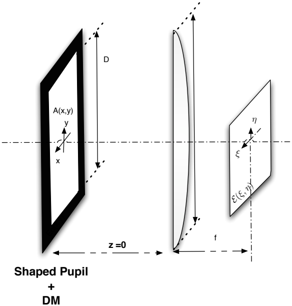
For this study we choose to use as a post apodiser a prolate function that provides a factor of 10^{-7} around $3\lambda/D$ and we look for a 2 DM induced apodisation that will bring the contrast down to 10^{-10} between IWA and OWA.

Questions regarding the surface of the DMs:
- **How to find the optimal shape $A_{2DM}(\tilde{x})$ that creates a Dark Zone between IWA and OWA?**
- **How does propagation effects influence that shape ?**
- **How to also control wavefront aberrations ?**

Question regarding the overall performance of this design:
-Does the 2 DMs setup provide an off-axis magnification.



Finding the optimal DM induced apodisation



- **We have a measurement of the field $E_{IM}(\xi)$ in the image plane**
- We are looking for apodisations of the form:

$$A_{2DM}(\tilde{x}) = 1 + \sum_{p=1}^{N} a_p f_p(\tilde{x}) \tag{6}$$

- The integrated intensity in the Dark Zone of the image plane is:

$$I_{DH} = \int_{DH} |E_{IM}(\xi) + \sum_{p=1}^{N} a_p (\widehat{A_{Post} f_p})(\xi)|^2 d\xi \tag{7}$$

Using the inner product introduced by Borde and Traub, rewriting $X = (a_1, ..., a_N)$ and choosing carefully the matrix M, the vector b and the scalar d this integrated energy can be rewritten as a quadratic form:

$$\boxed{I_{DH} = XMX^T + 2X.b^T + d} \tag{8}$$

Minimal apodisation algorithm

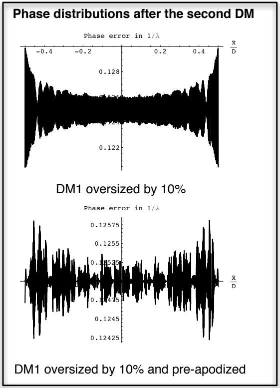
In order to find $A_{2DM}(\tilde{x})$ solve the quadratic programing problem: **Minimize $\sum a_p^2$ under the constraint that $I_{DH} < 10^{-10}$**

We used this algorithm in order to create a Dark Hole between a changing IWA and an OWA $= 10\lambda/D$.

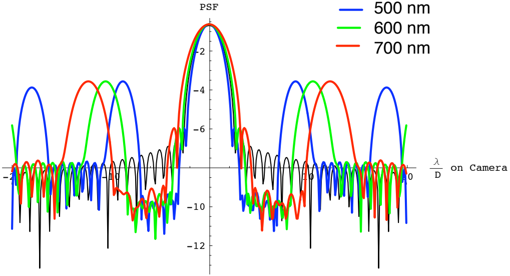
(MovieDiggingPSFs.avi)

When the IWA is too close the we find a solution that we choose to discard: $A_{2DM}(\tilde{x}) < 0$

We used our new semi analytical method to compute our propagation limited PSFs. In this case $x_0(\tilde{x})$ is always increasing, only remain edge effects that we mitigate using a pre-apodiser.



Propagated PSFs for $IWA = 3.8\lambda/D$. We used $D = 7$ cm and $z = 1$ m



Limitation: Our quadratic approximation is limited to apodisation with "small" third derivative. This means that it is only valid up to a certain spatial frequency, that was derived in the Fresnel case in Pueyo and Kasdin (2007):

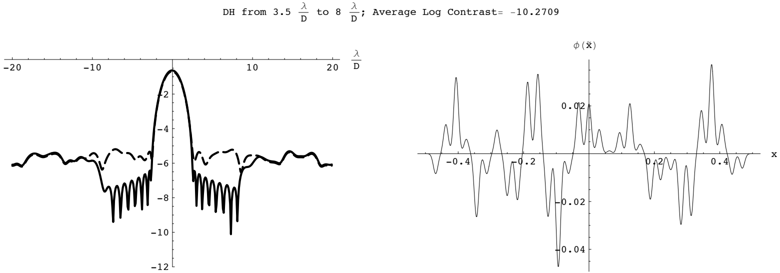
$$n_{lim} = \frac{D}{\sqrt{\lambda_0 Z}} \left(\frac{\lambda_0}{\Delta\lambda}\right)^{\frac{1}{4}} \quad (9)$$

Control Strategy

We assume now that $E_{IM}(\xi)$ also contains speckles due to amplitude and phase aberrations, it can be decomposed in two components:

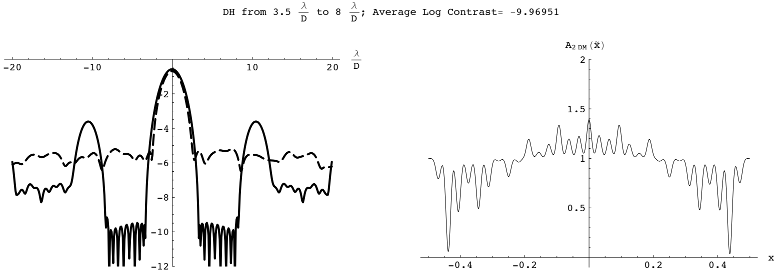
- $E_{IM}^{amp}(\xi)$ defined by $E_{IM}^{amp}(-\xi) = E_{IM}^{amp}(\xi)^\star$, created by pupil amplitude errors.
- $E_{IM}^{\phi}(\xi)$ defined by $E_{IM}^{\phi}(-\xi) = -E_{IM}^{\phi}(\xi)^\star$, created by pupil phase errors.

We first correct for the phase errors with the second DM using our algorithm

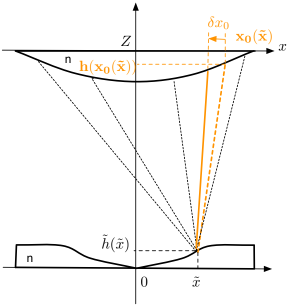


Control Strategy

We then correct for $E_{IM}^{amp}(\xi)$. The resulting $A_{2DM}(\tilde{x})$ compensates for both amplitude errors and the nominal PSF of the post apodiser.
The surface of each mirror is then computed using the differential equations of PIAA.



Off axis magnification



To study the off-axis distortion of the PSF of our system composed of two sequential DMs we apply this method with $\delta h(x) = \alpha x$, where α in the on-sky angular separation:

(OffAxisMovie.avi)

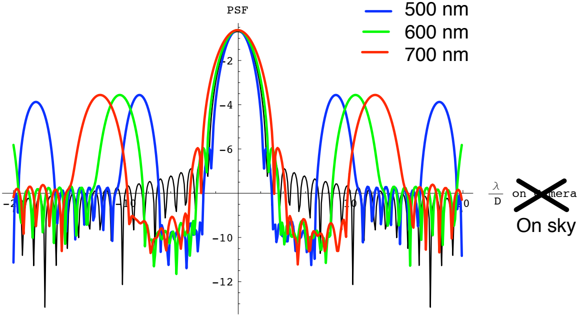
If the surface of the first DM does not completely satisfy the PIAA differential equation, that is $h(x) = h_0(x) + \delta h(x)$, then the position determined by ray optics is given by: $x_1(\tilde{x}) = x_0(\tilde{x}) + \delta x_0$, with:

$$\delta x_0 = \frac{-n\delta h'(x_0)}{\frac{1}{A(\tilde{x})Z)} - \delta h''(x_0)} \qquad (10)$$

We observe a magnification of a factor of 1.2

Conclusion

Propagation limited PSFs that take into account the magnification.



In this preliminary one dimensional analysis, we have designed an integrated coronagraph/wavefront control unit whose throughput is 30 percent and that can go as close in as $3.2\lambda/D$ under a 200 nm bandwidth.

A whole parameter space to be studied

This method will be generalized to two dimensions and carried out systematically in order to study the whole parameter space and provide a continuum of designs for missions such as TPF....stay tuned