1. Variation of insolation. 13 points.
Seasonal temperature changes are caused by the tilt of an object’s rotational axis (compared to the orbital plane). For example, at our approximate latitude of 40° on the Earth, the intensity of sunlight (insolation) on the surface varies by about 50% from winter to summer solstice.

For objects with significantly eccentric (non-circular) orbits, there can be an additional effect on the insolation because of the object’s variable distance from the Sun. The Technical Supplement describes ellipses, and here is some additional information about elliptical orbits:

![Diagram of an ellipse with foci labeled C, r_{peri}, and r_{ap}]

The distance from the center of the ellipse to the focus is \( c = e a \), where \( e \) is the eccentricity of the ellipse and \( a \) is the semimajor axis.

The Sun (or other central body) is at one of the foci of the elliptical orbit, and the closest and farthest approaches to the Sun along the orbit are denoted as \( r_{peri} \), the perihelion distance, and \( r_{ap} \), the aphelion distance.

You will also need to know that the insolation \( F \) as a function of distance can be given by:

\[
F = F_0 \left( \frac{r_0}{r} \right)^2,
\]

where \( F_0 \) is the solar constant, or 1370 watts m\(^{-2}\), \( r_0 \) is 1 AU, and \( r \) is the actual distance from the Sun to the object in units of AU.

A. Your Data Sheet gives the semimajor axis \( (a) \) and the eccentricity \( (e) \) for several orbits, but not the perihelion and aphelion distances. Create formulae that allow you to calculate \( r_{peri} \) and \( r_{ap} \) in terms of the known quantities \( a \) and \( e \).

B. Look up eccentricities and semimajor axes from the Data Sheet and list them in the first two columns of the table on the reverse side. Then calculate perihelion and aphelion distances.

C. Calculate the insolation for each body at perihelion and aphelion and list them in the table. The ratio of the perihelion and aphelion insolation gives you the annual variation for each body; list this in the last column.

D. When comets are visible to the naked eye, it is because large quantities of volatiles are being sublimated away by solar heating. Use Kepler’s second law, along with the information in the table below, to explain why comets have managed to survive for the age of the solar system without losing all their volatiles.
Object | Eccentricity $e$ | Semimajor axis (AU) | Perihelion distance (AU) | Aphelion distance (AU) | Perihelion insolation (W m$^{-2}$) | Aphelion insolation (W m$^{-2}$) | Annual variation in insolation (%)
---|---|---|---|---|---|---|---
Mercury | | | | | | | |
Earth | | | | | | | |
Mars | | | | | | | |
Jupiter | | | | | | | |
Pluto | | | | | | | |
Comet Halley | 0.967 | 17.9 | | | | | |

3. Measuring temperature. 7 points.
In a certain laboratory, there are two cats. Infrared spectra of the cats have been taken. You will use the spectra to determine the temperatures of the cats, and determine whether each cat is alive or dead.

A. The table below contains the measurements of the cats’ infrared intensity. Plot the measurements on the graph below, and represent the uncertainties as error bars. Use squares for Cat 1, and circles for Cat 2. If you picked up your problem set in class, the first Cat 1 point is done for you.

<table>
<thead>
<tr>
<th>Wavelength (µm)</th>
<th>Cat 1 intensity</th>
<th>Uncertainty of Cat 1 intensity</th>
<th>Cat 2 intensity</th>
<th>Uncertainty of Cat 2 intensity</th>
</tr>
</thead>
<tbody>
<tr>
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<td>8.34</td>
<td>0.02</td>
<td>9.45</td>
<td>0.17</td>
</tr>
<tr>
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<td>0.07</td>
<td>9.68</td>
<td>0.05</td>
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<td>0.00</td>
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<tr>
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<td>9.59</td>
<td>0.19</td>
<td>8.78</td>
<td>0.03</td>
</tr>
</tbody>
</table>

B. Use Wein’s Displacement law to estimate the temperatures of the two cats. If you are a permanent resident of the United States, convert the cat temperatures to degrees Fahrenheit. Otherwise, convert the temperatures to Celsius. Then, state whether each cat is likely to be alive or dead.