Poisson model

For CMB fluctuations it's easy to know how to relate theory, $C(r)$, to observations at JTA.

How do we connect

$$S(P) = < S(2P) S(2) > \text{ vs. } S_{PV} = \bar{S} S_{PV} (1 + \delta_{PV})$$

for galaxies?

Introduce a model to allow us to go from its fields to point processes.

This model adds stochasticity, makes assumptions and is only approximate.

Poisson model (Longair, Limber)

First select $p(P)$ from ensemble

Place an object with probability $SP = p(P) S_{P}$, independent of other objects.

Note: This can't be whole story - deviations due to volume exclusion (hard spheres)
or excess clustering.

A more clever start formulation can be found in "Theory of point processes" (Daley & Vere-Jones)
Non-Poisson from N-body see Cen, Memono et al. [arXiv:050608]

Note that this model does what we want.

$$SP = p(P) S_{P}(p(P)) S_{P}$$

average over $p$: $< p(P) p(x > r) > = \bar{p}^{2} (1 + \bar{S}(r))$ so

$$SP = \bar{S}^{2} S_{P} S_{V} (1 + \bar{S}(r))$$
Now when we compute 2-point神圣 there are 2 levels of averaging

\[ \langle n, n_j \rangle = \frac{\langle n + h_5, n(h_5) \rangle}{n} \quad \text{for \textbf{c} in cell \textbf{i} j} \]
\[ = \frac{\langle n(t + h_5), n(h_5) \rangle}{n} + \delta_{ij} \frac{\langle n(h_5) \rangle}{n} \]
\[ = n^2 (1 + 3j) + \bar{n} \delta_{ij} \]

Let's look at the 1st step again. Can use

\[ p(n; \mu) = \frac{\mu^n}{n!} e^{-\mu} \]
\[ \langle n \rangle = 1 \]
\[ \langle n^2 \rangle = \mu \]
\[ \langle n(n-1) \rangle = \mu^2 \quad \Rightarrow \quad \langle n^2 \rangle = \mu + \mu^2 \]
\[ \langle n(n-1)(n-2) \rangle = \mu^3 \quad \Rightarrow \quad \langle n^3 \rangle = \mu + 3\mu^2 + \mu^3 \]
\[ \text{etc.} \]
\[ \text{along with independence: } \langle n, n_j \rangle = \langle n \rangle \langle n_j \rangle \quad \text{if} \quad \text{i} \neq \text{j} \]

But there is an easier way to do it.
Divide space into infinitesimal cells with \( \mu < 1 \). Then \( n_i = 0 \) or 1 so
\[ \langle n \rangle = \langle n^2 \rangle = \langle n^3 \rangle = \ldots \]

So for example

\[ \langle n \rangle = \frac{1}{\sigma^2_k} \left[ \int d^3 r_1 d^3 r_2 \left( \frac{1}{n^2} \right)^\mu \left( \frac{n_1 - \bar{n}}{n} \right) \left( \frac{n_2 - \bar{n}}{n} \right) \right] \]
\[ = \left\{ \begin{array}{ll}
\sum \delta_{ij} \frac{N_i - N}{N} \left( \frac{N_j - N}{N} \right) & \text{if } \mu = 0,
\end{array} \right. \]
\[ = \sum \frac{N_i - N}{N} \left( \frac{N_j - N}{N} \right) \left( \frac{S_{ij} M_i + S_{ij} M_j}{\frac{N}{n}} \right) \]
\[ = \int d^3 \bar{r}_1 d^3 \bar{r}_2 \left( \frac{1}{n^2} \right)^\mu \left( \frac{S_{ij}}{\frac{N}{n}} \right) \left( \frac{S_{ij}}{\frac{N_j}{n}} \right) \bar{r}_i \bar{r}_j \frac{1}{\varphi} \]
\[ = p(k) + \frac{1}{\varphi} \]
Poisson model

Note we have two ridge causes of variance, power spectra add and white/Poisson noise has $F(k) = \text{const.}$

So our estimate of power would be

$$\hat{P}(k) = 18_k l^2 - \frac{1}{n}$$

How would we compute $SP$?
Assume linear theory with $\delta_i$ independent Gaussian.
In the limit $n \to \infty$

$$\text{Var}[\hat{P}] = \langle \hat{P}^2 \rangle - \langle \hat{P} \rangle^2$$
$$\quad = \langle \delta_i^4 \rangle - \langle \delta_i^2 \rangle^2$$
$$\quad = 2P(k)$$

$$\frac{SP}{P} = \sqrt{2}$$ for $N$ independent modes.

Now note finite $n \approx \infty$. If Poisson "noise" is independent of signal

$$\langle (s+n)(s+n) \rangle - \langle s+n \rangle^2 = \langle s^2 \rangle - \langle s \rangle^2 + \langle n^2 \rangle - \langle n \rangle^2 \quad \text{if} \quad \langle s \rangle = 0$$

The "$s$" terms gave $2P(k)$ while the "$n$" terms gave $2 \frac{1}{n} \to 0$

$$\frac{SP}{P} = \sqrt{\frac{2}{N}} \left[ 1 + \frac{1}{NP} \right]$$

to leading order in $\frac{1}{n}$

(yield result in Markovič + White MNRAS '99).
Another way of looking at this result: what we measure is random with power $P(k) + \frac{1}{n}$.

Subtracting a constant from $B$ doesn't change statistics so

$$\frac{\hat{S}^2}{B} = \frac{S^2}{B} = \sqrt{1 + \frac{1}{n}}$$

Note: Have "sample variance" and never contributions to $S^2$.
First is multiplicative and pushes to maximize sky area \textit{(number of modes)} while shot-noise is "additive".

Once $nP > 1$ reach diminishing returns for $S^2$. At $nP = 5$ are 80% to $\infty$.
Near peak of $P(k)$ have $P = 2500 - 5000 (hMpc^{-1})^3$ so optimal $n \approx 10^{-4}$, or 6 less than $L_x$ galaxies.

Kaiser - subsample and cover 8% of area.

\textit{BUT must subsample randomly}...