An Introduction to the Standard Model: Physics 221a Section Lecture Notes

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This is a brief summary of the Standard Model of particle physics, with an emphasis on understanding the underlying assumptions that go into it. It is important to understand which aspects follow from general principles, which aspects are put in by hand, and which of these are the most important shortcomings. There is a lot here, and I imagine that to some this is all new and confusing. My hope for these people is that you learn the general structure of the particles that make up the world around us. For those with a little more familiarity with particle physics, I hope this will help you to begin to understand not just *what* the "fundamental" particles and interactions are, but *why* nature might behave this way.

1 Lorentz Invariance

The first input into the Standard Model is Lorentz invariance, which has many consequences, but in particular that nature should be described by a local, scalar Lagrangian constructed out of representations of the Lorentz group SO(3,1). This says that the particles of nature must transform in a nice way under Lorentz transformations, and that their dynamics must be governed by a Lorentz-invariant action. The representations of SO(3,1) are quite similar to the representations of SO(3) that are familiar to us, so there are scalars, spinors, vectors and so on, but there is some subtly. The most important difference is that spin-$\frac{1}{2}$ fermions are described by a four-component Dirac spinor ψ which describes both a particle and its antiparticle. A Dirac spinor in four dimensions is equivalent to either two Weyl spinors, one left- and one right-handed, or to two real Majorana spinors.

From all these representations, we are left with a Lagrangian of the form

$$\mathcal{L} = \Sigma c_{i...j} \phi^i ... \phi^j$$

where ϕ denotes all the possible fields in the theory (and their derivatives), and the $c_{i...j}$ denotes both a kind of Klebsch-Gordon coefficient to make a scalar out of the component fields, and the appropriate coupling constant.

2 Renormalization

The next important input into the theory is that it must be quantum mechanical, which in turn brings about the notion of renormalization. Renormalization is a complicated topic to understand in detail, but it is mostly about *scaling*: how do things change with energy scale? To really understand this, we would need to understand the quantum effects such as running of coupling constants, but it turns out that a lot of renormalization can be understood just from looking at dimensional analysis. We will work in units of $\hbar = c = 1$ in which any quantity may be assigned a single dimension, its mass-dimension. $[\mathrm{E}] = [\mathrm{m}] = [1/\mathrm{L}] = 1$.

Since the action S= $\int d^4x\mathcal{L}$ must be dimensionless (it appears in the exponent in the path-integral), the Lagrangian must have dimension 4 to cancel that of d^4x. Related to the fact that bosons have two derivatives in their kinetic terms and fermions one, these fields have a canonical dimension [boson]=1 and [fermion]= 3/2. If the Lagrangian contains some composite operator $\mathcal{O}_i$ with coefficient g_i, dimensional analysis requires that $g_i \sim \Lambda^{4-d_i}$ where Λ is the fundamental scale in the theory, to be taken near the Planck scale. We see that if $d_i = [\mathcal{O}_i] > 4$ the coupling constant g_i is naturally suppressed by a very large number, and *we need not consider such a term in our Lagrangian.*

Consideration of renormalization naturally eliminates all but a small number of possible interactions (the "Renormalizable" terms), leaving the kinetic terms and only the following Lorentz-invariant interactions constructed out of scalars ϕ, spinors ψ, vectors A_μ, and a set of 4×4 matrices γ^μ analogous to the Pauli spin matrices:

$$\begin{array}{ccc} \phi^n & (n \leq 4) & \\ \bar{\psi}\gamma^\mu\psi A_\mu & \bar{\psi}\psi & \bar{\psi}\psi\phi \\ A_\mu A^\mu \phi^2 & (A_\mu A^\mu)^n & (n \leq 2) \end{array}$$

This is a dramatic simplification, but we can actually go further. Renormalization tells us that the vector field mass term, $A_\mu A^\mu$, must either have a coefficient $\sim \mathrm{M}_{pl}^2$, or must vanish. Since a Planck-mass particle would be impossible to produce at our energy scales, any vector field in the Standard Model must be massless. This is an extremely important result, because massless vector fields have only two states (helicities or polarizations) rather that the three $\mathrm{S}_z = \pm 1, 0$ states of the massive vector particle. In order to remove this extra degree of freedom, any fundamental theory with vector particles *must have a local gauge symmetry*, $A_\mu \to A_\mu + \partial_\mu \lambda$, which can absorb the unphysical degree of freedom. This is why gauge invariance plays such an important role in the Standard Model. It eliminates the $(A_\mu)^n$ terms and imposes strong relationships among the remaining terms in the action.

3 The Standard Model

We have seen that Lorentz invariance and Renormalization dramatically constrain the content of the Standard Model to be scalars, fermions, and gauge bosons interacting according to a limited number of Lorentz-invariant, renormalizable, and gauge-invariant interaction terms. To specify the Standard Model, we must choose the matter content and gauge group to match that observed experimentally. The gauge group is G=SU(3)×SU(2)×U(1) with gauge bosons G_μ^a, W_μ^i and B_μ respectively. This group couples to three generations of matter consisting of five types of Weyl spinor "qudle" and to the Higgs boson ϕ according to the following representations:

$$\begin{array}{rcl} \text{Group} & : & 3\ \ 2\ \ 1 \\ q_L^i & : & (\mathbf{3}, \mathbf{2}, \frac{1}{6}) \\ u_R^i & : & (\mathbf{3}, \mathbf{1}, \frac{2}{3}) \\ d_R^i & : & (\mathbf{3}, \mathbf{1}, -\frac{1}{3}) \\ l_L^i & : & (\mathbf{1}, \mathbf{2}, -\frac{1}{2}) \\ e_R^i & : & (\mathbf{1}, \mathbf{1}, -1) \\ & & \\ \phi & : & (\mathbf{1}, \mathbf{2}, \frac{1}{2}) \end{array}$$

We see that the quarks $q_L = (u_L,\ d_L)$, u_r and d_R are the only "colored" matter particles, coming in the red, green and blue of the SU(3) **3** representation, and that only the left-handed fermions q_L and the lepton $l_L = (\nu_L,\ e_L)$ feel the weak force SU(2), while their right-handed counterparts do not.

The index i=1,2,3 on the fermions represents the generation, or flavor. For example e^1 is the electron, while $e^2 = \mu$ is the muon and $e^3 = \tau$, the heavier, but otherwise identical cousins of the electrons.

The most general Lagrangian that we can write down that has the above particle content and gauge symmetries is the Standard Model:

$$\begin{aligned} \mathcal{L} = & - \frac{1}{4}\sum_{n=1}^{3} tr(\mathbf{F}_{\mu\nu}^{(n)}\mathbf{F}^{\mu\nu}_{(n)}) + \sum_{f=qudle\times 3} \bar{\psi}^{(f)}\gamma^\mu D_\mu \psi^{(f)} + |D_\mu \phi|^2 \\ & - (\lambda_e)_{ij}\bar{e}_L^i \cdot \phi e_R^j - (\lambda_d)_{ij}\bar{q}_L^i \cdot \phi d_R^j - (\lambda_u)_{ij}\epsilon^{ab}\bar{q}_{La}^i \phi_b^\dagger u_R^j + h.c. \\ & + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger\phi)^2, \end{aligned}$$

where $D_\mu = \partial_\mu - i\sum_{n=1}^3 g^{(n)}\mathbf{A}_\mu^{(n)}$ is the covariant derivative, and

$$\begin{aligned} \mathbf{F}_{\mu\nu}^{(n)} & = \partial_\mu \mathbf{A}_\nu^{(n)} - \partial_\nu \mathbf{A}_\mu^{(n)} + [\mathbf{A}_\mu^{(n)}, \mathbf{A}_\nu^{(n)}] \\ & = F_{\mu\nu}^{a\ (n)} \mathbf{T}_a^{(n)} \end{aligned}$$

is the matrix-valued field-strength tensor for the gauge field corresponding to SU(n) (n=2,3) or the hypercharge U(1) (n=1), and $g^{(n)}$ is the coupling constant appropriate to each gauge group. The $\mathbf{T}_a^{(n)}$ are the appropriate representations of the generators of each group, a diagonal matrix $\mathbf{Y}$ in the case of hypercharge.

This may look very complicated, but the equations generated from this (plus the Einstein-Hilbert action $\mathcal{L} = \sqrt{|g|}\mathcal{R}$) reproduce *every experiment ever conducted* (prior to 1998; see below), including some with twelve-digit accuracy. It is amazing that so few assumptions could lead to a theory from which all else can in principle be derived. It is surely a pinnacle of human achievement.

Let's look at it a bit closer. The first line contains the covariant kinetic terms for the gauge particles, fermions and Higgs boson respectively. The next line is the Yukawa couplings $\sim \phi\bar{\psi}\psi$ between the Higgs ϕ and the qudle fermions. The $\lambda_{e,u,d}$ are 3×3 matrices of coupling constants with indeces i, j running over the three generations of "flavor space". The last term is the Higgs potential $-V(\phi)$ with *negative* quadratic term. Note in particular that SU(2) gauge-invariance has prohibited fermion mass terms, which in terms of Weyl spinors would look like $m(\bar{e}_L e_R + h.c.)$. In fact, there is only a single dimensionful parameter in the whole standard model, μ^2.

3.1 Spontaneous Symmetry Breaking

All the masses of the Standard Model are generated by spontaneous symmetry breaking. The Higgs' potential drives it to acquire an expectation value which feeds a mass to all the fermions through the Yukawa terms. Replace ϕ with $\langle\phi\rangle$ and the Yukawa terms become mass terms that are otherwise not allowed until SU(2) is broken. The gauge bosons W^1_μ, W^2_μ and $Z_\mu = (g^{(2)}W_\mu - g^{(1)}B_\mu)/\sqrt{g^2_{(1)} + g^2_{(2)}}$ become massive through the $\sim \mathbf{A}_\mu \mathbf{A}^\mu \langle\phi\rangle^2$ term in the Higgs' kinetic term. The other linear combination

$$A_\mu = (g^{(1)}W_\mu + g^{(2)}B_\mu)/\sqrt{g^2_{(1)} + g^2_{(2)}}$$

remains massless and is the photon we know and love, which is the gauge boson of U(1) left unbroken called "charge" with the diagonal generator $\mathbf{Q} = \mathbf{T}^3 + \mathbf{Y}$.

3.2 Global Symmetries: Baryon and Lepton Number, Flavor and CP

The above action happens to have, or almost have, a number of global symmetries that we didn't put in by hand. Among these are baryon and lepton number, corresponding to a phase rotation of all quarks together, or all leptons (electrons and neutrinos) together. Baryon number conservation is what protects the proton from decay to a lighter lepton, and it is a nice accident that it emerges on its own. Two other global symmetries, flavor conservation and CP (simultaneous Charge and Parity reversal) are almost conserved, but not quite. As we have written it, the Yukawa couplings, and thus the fermion mass matrices, are not diagonal in flavor space. We should make a change of basis, mixing the generations, to diagonalize the mass matrix. This change of basis leaves most of the Lagrangian invariant, but it leaves its footprints in the coupling of fermions to the weak force, introducing flavor-changing and CP-violating CKM matrices. The BaBar experiment currently under way at SLAC is currently confirming that all the CP violation observable in nature indeed comes from these CKM matrices.

4 Shortcomings of the Standard Model: Good news and bad news

Now let us reexamine some of the assumptions that brought us to this model. We'll see that some of what looked especially arbitrary is not, and some things that looked harmless are actually very problematic.

4.1 The θ-Term

First, it should be noted that there is actually one additional object that could be added to the above Lagrangian, the "θ-term", of the form $\mathcal{L}_\theta = \frac{\theta}{8\pi^2}\epsilon^{\mu\nu\alpha\beta} tr(\mathbf{F}_{\mu\nu}\mathbf{F}_{\alpha\beta})$, which is omitted by hand. This is somewhat reasonable because it can be written as a total derivative, $\mathcal{L}_\theta = \partial_\mu \omega^\mu_{cs}$ for a particular one-form ω^μ_{cs}, and so only enters the action as a boundary term. In fact, topological defects called instantons *do* make this term relevant. Explaining its absence is related to an outstanding problem called the strong-CP problem, for which hypothetical pseudo-scalar particles called axions present one solution.

4.2 Charge Assignments and Anomalies

It turns out that gauge invariance places an additional constraint on the Lagrangian that actually makes a lot of the arbitrary-seeming particle content of the Standard Model a lot more natural. This constraint is called anomaly cancellation. The Lagrangian we have constructed is gauge invariant at the classical level, but it is a very subtle question whether this gauge invariance will be preserved upon quantization. Quantum mechanical violations of symmetries are called anomalies, and when anomalies violate a *gauge* symmetry, the theory is considered terminally ill, because the gauge symmetry needed to remove the unphysical $S_z = 0$ polarization of the gauge particles is absent, and unitarity is violated.

Analysis of anomalies is a bit technical, but it turns out that anomalies cancel automatically in two cases: when the theory is parity symmetric, and when the gauge group is SU(2). This fits perfectly with the fact that the SU(3) couplings are parity symmetric, and that it is SU(2) whose couplings are totally parity violating, making a lot of the above assignments a lot more plausible. But in order for the full theory to be anomaly-free, there are four different conditions that must be satisfied by the hypercharges. It turns out that the apparently random assignments of hypercharge Y above are actually one of the simplest solutions to these conditions, with cancellations occurring between the different particles in each generation. Anomalies by no means pick out the Standard Model particle content uniquely, but they make it a lot less arbitrary. In particular, anomalies require that whatever the particle content, it must come in generations.

4.3 Why Three Generations?

Our existence is dependent on CP violation, in order to avoid annihilation of all the particles and antiparticles of the early universe. All the CP violation that is observed comes from the flavor rotation matrices. If there were two or fewer generations, the rotation matrices would be simpler and this CP violation would be absent. This is an unpleasantly anthropic and incomplete explanation for the number of generations, but still an interesting fact worth knowing.

4.4 The Fine-Tuning Problem

There is actually one particularly big cheat in the above Lagrangian. We argued away explicit mass terms of vector bosons based on renormalization arguments, but we kept the Higgs boson mass term μ^2 that sets the scale of all other physics. Renormalization tells us that quantum corrections should generate a Planck-scale mass for this, pushing the entire theory 15 orders of magnitude higher. We force this theory to make sense by tuning the Planck-scale "bare" mass to precisely cancel these quantum corrections, so that the physical mass is something like

$$M_h^2 = 1.0000000000000000000000000000001 \times 10^{36}\mathrm{GeV}^2 - M_{pl}^2.$$

From this point of view, the Higgs mass had to be chosen very precisely in order to give any physics below the Planck scale at all. This is clearly unacceptable, but fortunately has a solution: supersymmetry protects the Higgs mass against severe quantum corrections and eliminates the need for fine-tuning. I will leave to next week (?) a discussion of supersymmetry, but the fine-tuning problem is one of the greatest reasons to think that supersymmetry must exist near the electroweak scale.

4.5 Why the Split Among the Generations?

Why are the eigenvalues of the mass matrices so different from each other, which is to say, why is there a splitting of the masses of the three generations by factors of $\sim$100? Note that this is different from wondering why quarks are so much heavier than leptons, which can be explained by the fact that quarks couple to the strong force and so receive stronger quantum corrections to their masses. There is no single widely accepted solution to this problem, but it is a question of only a couple of orders of magnitude. It's something people think about, but don't fret about. On the other hand, people do worry about

4.6 The Hierarchy Problem

Why is the electroweak scale so far below the Planck scale? Why is there a gap of fifteen orders of magnitude between what is thought of as the fundamental scale of physics, $M_{pl} \simeq 10^{18} GeV$, and the scale of most of our physics $M_{EW} \simeq 100 GeV$? This issue is independent of the fine tuning problem above, and isn't solved by supersymmetry. There are a lot of different ideas on it, including the extra-dimensions solutions of ADD and Randall-Sundrum that we mentioned in a past lecture, but these both have problems of their own. A rough outline of a solution that strikes me as plausible comes out of the fact that renormalization tells us that parameters in the Lagrangian "run" with energy scale, often logarithmically. It is plausible that a mass parameter originally on the Planck scale slowly runs lower and lower. Over many orders of magnitude, this running could push it negative, triggering supersymmetry breaking followed by electroweak symmetry breaking and the generation of the mass scales central to our physics. But understanding supersymmetry breaking remains one of the biggest problems in phenomenology today.

4.7 Neutrino masses

We stated above that the Standard Model explains every experimental result up to 1998. In 1998, neutrino oscillation experiments confirmed that neutrinos must have very small masses, likely on the order of an eV. This requires the addition of three extra particles $\nu_{R\ e,\mu,\tau}$ to the content of the Standard Model, because massive particles need both left- and right-handed components to fill out the spin-up and spin-down states. The presence of these states actually makes the Standard Model even a bit cleaner, as it fills a hole: now there are the same number of quark and lepton states. There is even a really nice explanation of such a small neutrino mass known as the see-saw mechanism, which takes advantage of the fact that right-handed neutrinos are totally uncharged, and therefore allow what is called a Majorana mass term, prohibited by gauge invariance for every other particle.