1 Polarimeters : Instrumentation Techniques

In principle measurement of all four Stokes parameters I,Q,U,V will give us complete information on the polarization state of a partially polarized wave (Born and Wolf 1959). The most general way to do this involves using a retarder and/or polarizer in series (Clarke and Grainger 1971). This is the priciple behind polarimeters at IR (Platt et al. 1994), submm (Dragovan 1986) and mm (Clemens et al. 1990). Polarimeters have been constructed for a wide range of wavelengths ranging from the optical through the infrared and submillimeter to the millimeter. However one must keep in mind that the polarimeters referred to in the previous section have all been built for single dish telescopes. The techniques used in polarimetric interferometry will be dealt with in the next section.

1.1 Polarization Response of an Interferometer

The additive nature of the Stokes parameters makes them very suitable for describing the polarization of an arbitrarily polarized source (Chandrasekhar 1950). The Stokes parameters are defined as

$$\begin{aligned} I &= \langle (E_l{}^o)^2 \rangle + \langle (E_r{}^o)^2 \rangle \\ Q &= \langle (E_l{}^o)^2 \rangle - \langle (E_r{}^o)^2 \rangle \\ U &= 2\langle E_l{}^o E_r{}^o \rangle \cos\delta \\ V &= 2\langle E_l{}^o E_r{}^o \rangle \sin\delta \end{aligned} \tag{1}$$

where the subscripts denote the directions (left or right) and δ is the phase difference between these components. The total flux is given by I, linearly polarized radiation is specified by the parameters Q and U, while V gives the circular polarization. The linear polarization components, Q and U, vary under rotation as the real and imaginary parts of the complex quantity P, whose magnitude m gives the fraction of linear polarization and whose phase is twice the position angle of the plane of polarization of the electric field vector (2χ).

$$P = Q + iU = \sqrt{Q^2 + U^2}\, e^{i\,tan^{-1}(U/Q)} = mIe^{i2\chi} \tag{2}$$

where I is the total intensity, m is the percentage of linear polarization ($= \sqrt{Q^2 + U^2}/I$) and χ is the position angle of the plane of polarization. Morris *et al.* (1964) show that the polarization distribution of a radio source can be described by the distribution of the four Stokes parameters over the surface. Therefore, if we assume that the polarization of the incoming radiation does not change over the bandwidth of observation, polarization properties of the source can be obtained by measurement of the Fourier transform of the four Stokes parameters followed by a Fourier inversion. If the interferometer had a feed sensitive to only one of the four parameters, then the output would be a function of only the distribution of that parameter. In general that is not the case, and a typical interferometer responds to at least two of the four parameters. In the following treatment of the interferometer response, the additive nature of the parameters is implied.

Let us now consider an interferometer made of two antennas indicated by subscripts m and n. The feeds at each antenna can be described in terms of the axial ratio and the orientation of the major axes of the polarization ellipses. Let the polarizations of the two antennas be denoted by ψ_m, χ_m and ψ_n, χ_n, where ψ denotes the orientation of the major axis of the polarization ellipse with respect to the coordinate system defining the Stokes parameters and χ denotes the ellipticity given by $\tan\chi = b/a$, the axial ratio. The response to the four Stokes parameters I,Q,U,V can be shown to be (Morris *et al.* 1964)

$$
\begin{aligned}
R_{mn} = \; & \frac{1}{2}k\{I[\cos(\psi_m - \psi_n)\cos(\chi_m - \chi_n) + i\sin(\psi_m - \psi_n)\sin(\chi_m + \chi_n)] \\
& + Q[\cos(\psi_m + \psi_n)\cos(\chi_m + \chi_n) + i\sin(\psi_m + \psi_n)\sin(\chi_m - \chi_n)] \\
& + U[\sin(\psi_m + \psi_n)\cos(\chi_m + \chi_n) - i\cos(\psi_m + \psi_n)\sin(\chi_m - \chi_n)] \\
& - V[\cos(\psi_m - \psi_n)\sin(\chi_m + \chi_n) + i\sin(\psi_m - \psi_n)\cos(\chi_m - \chi_n)]\}
\end{aligned} \tag{3}
$$

where k is a factor related to the gains and the effective apertures. For the case of identically circularly polarized feeds, we have $\psi_m = \psi_n = \psi$ and $\chi_m = \chi_n = \chi$, where χ takes values of $\pi/4$ or $\pi/4$ for left and right circular polarizations respectively. We then get

$$R_{LL} = \frac{1}{2}k(I - V) \tag{4}$$

$$R_{RR} = \frac{1}{2}k(I + V) \tag{5}$$

If however the feeds are oppositely circularly polarized, then $\chi_m = -\chi_n = \chi$ and $\psi_L = \psi_R + \pi/2$. The response, we get is

$$R_{RL} = \frac{1}{2}k(+U - jQ)e^{-i2\psi_m} \tag{6}$$

$$R_{LR} = \frac{1}{2}k(-U - jQ)e^{+i2\psi_m} \tag{7}$$

Further, if we assume that the source has no circular polarization ($V = 0$), then the response to indentical feeds is just I. Also, from linear combinations of the last two equations and choice of ψ_R, it is possible to obtain Q and U which are the linear polarization components. For example, under the choice of $\psi_R = 0$, the sum and difference of the last two responses will give us the linear polarization U and Q respectively.

$$I = 2\frac{R_{LL}}{k} = 2\frac{R_{RR}}{k} \tag{8}$$

$$U = \frac{R_{RL} + R_{LR}}{k} \tag{9}$$

$$Q = \frac{R_{LR} - R_{RL}}{ik} \tag{10}$$

Once we have obtained the Stokes parameters we can obtain the degree of linear polarization and also the polarization angle.

$$m = \text{percent of linear polarization} = \frac{\sqrt{Q^2 + U^2}}{I} \tag{11}$$

$$\chi = \frac{1}{2}\tan^{-1}(U/Q) \tag{12}$$

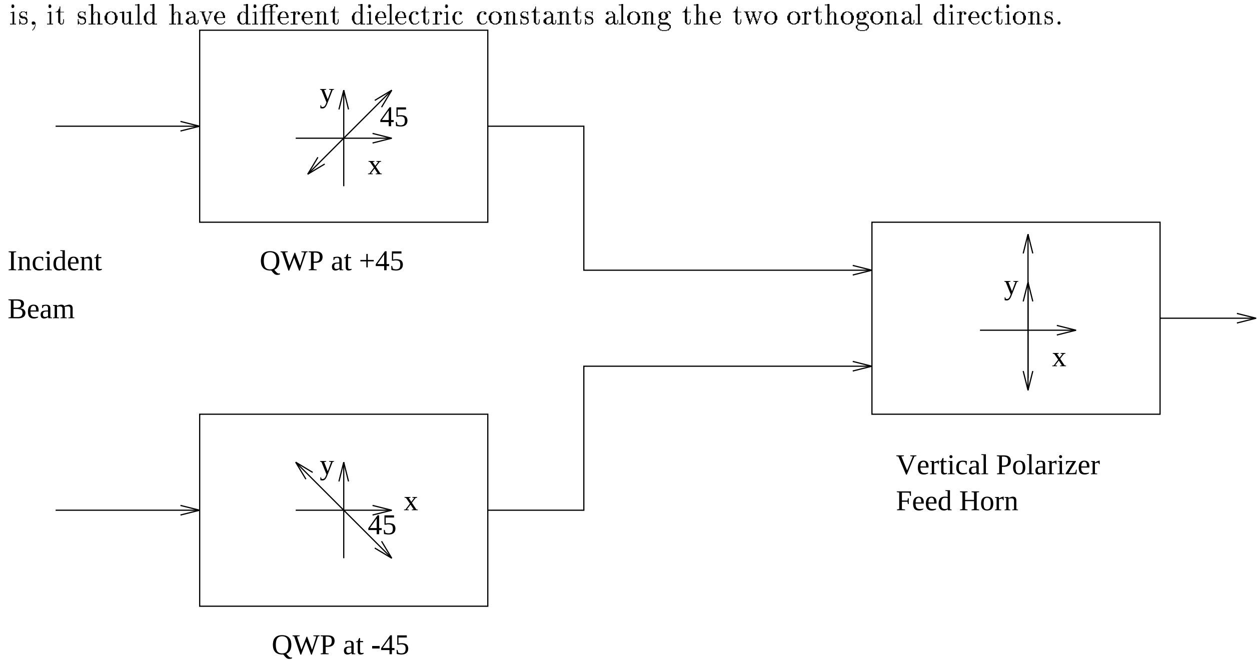
The above results have been derived for the case of an unresolved source. If the source is extended, the Stokes parameters (I,Q,U,V) can be written as functions of the angular coordinates (ξ, η) across the source, that is, I(ξ, η), Q(ξ, η), U(ξ, η) and V(ξ, η). The outputs of the interferometers are thus related to the two-dimensional Fourier transforms of the functions and can be written as i(x,y), q(x,y), u(x,y) and v(x,y).

$$I, Q, U, V\,(\xi, \eta) \overset{\mathrm{FT}}{\Longleftrightarrow} i, q, u, v\,(x, y) \tag{13}$$

In principle, we can invert these four transforms to obtain the source distributions.

1.2 Polarizers at millimeter wavelengths

The function of the polarimeter will be to convert linearly polarized radiation to one that is circularly polarized. A block diagram of the polarimetry system is shown in the accompanying figure. The active element in this scheme is a quarter-wave plate. A quarter-wave plate is a device that gives a retardation of 90^o between two mutually orthogonal components of an electromagnetic wave. If the incident components have the same amplitude and phase, then the circular polarizer retards the phase of one of the components by 90^o. It therefore converts the linearly polarized to circularly polarized radiation. A review of existing quarter-wave plate designs is presented by van Vliet and de Graauw (1981). The exact details of the working of a quarter-wave plate are given in the appendix. In order to produce circular polarization, the material needs to be anisotropic, that is, it should have different dielectric constants along the two orthogonal directions.



An analysis using the Jones vector methods (Appendix **) shows that in the two cases (QWP at +45 and -45 followed by a vertical linear polarizer, we get either the left or the right circularly polarized component. As derived in a preceding section, correlations of these can yield us the Stokes parameters for linear polarization Q and U.

The anisotropy is produced by making a number of deep grooves in a slab of a dielectric material like rexolite. The spacings between the grooves are set equal to the depth of the grooves. An incoming linearly polarized wave sees different dielectric constants for the components along the "fast" and "slow" axes of the plate, perpendicular and parallel to the grooves respectively. This depth depends on the frequncy of observation and the dielectric constant of the material being used.The thickness is adjusted so that when the radiation emerges from the plate the two orthogonal polarizations have a phase shift of 90^o. The thickness of the plate is given by

$$d = \frac{\lambda_o}{4\left(\sqrt{\epsilon_y} - \sqrt{\epsilon_x}\right)} \tag{14}$$

where ϵ_y and ϵ_x are the relative dielectric constants along the orthogonal directions which are given by Kirschbaum and Chen (1957).

$$\epsilon_y = \frac{1+\epsilon}{2} \tag{15}$$

$$\epsilon_x = \frac{2\,\epsilon}{1+\epsilon} \tag{16}$$

where ϵ is the value of the dielectric constant of the material being used. For the above formulae to work, the widths of the grooves and slots should be less than $\lambda_o/3$.

A number of factors need to be taken into account in the process of choosing and designing a quarter wave plate. Among the first is the ease with which the plate can be manufactured. Another important consideration is the bandwidth of the circular polarizer. In theory, a quarter-wave plate works perfectly only at one wavelength and at other wavelengths it tends to produce elliptically polarized radiation. The bandwidth can be defined as the frequency range over which the ellipticity is less than than 10%. In addition, there will be losses due to reflection and absortion within the device. These losses can be determined only when the complex dielectric constant has been determined. These can vary depending on the way the plate has been made. Yet another factor that needs to be considered is that the beam that is incident on the quarter-wave plate is not plain but has a spherical wavefront. This will introduce a further inefficiency in the action of the polarizer. All of the above effects needs to be calibrated before a polarization measurement can be made.

The optical design of the receiving system at each antenna is described by Lugten *et.al.* (199*). The measurement of polarization requires a set of two polarization plates for each frequency band. The difference in the polarizing properties of the two plates arising from the difference in physical properties like thickness can introduce additional complications which need to be removed by a suitable calibration. The level of polarization due to instrumental effects needs to be characterized and calibrated out.

2 Sensitivity Calculations

I am calculating the sensitivity assuming that I will have a ten element interfe rometer available (hopefully) at a wavelength of 1.3 mm. The interferometer configuratio n will be such that half the antennae will be observing in one polarization and the oth er half will be observing the orthogonal polarization. We therefore have 5 antennae observing LCP and 5 observing RCP. By correlating identical polarizations we get a measure of the total intensity I and by correlating the orthogonal polarizations we get a result which is a linear combination of the Stokes parameters Q and U. In addition we need to flip the polarizations for half the time. This is necessary if we want to resolve the linear polarization observations into Q and U. From the above configurations there are 25 baselines observing Q,U and 20 baselines observing I. We can now obtain the required sensitivities. From Equation 10.36 on p277 of your article Interfero. (Verschuur 1974)

$$\Delta T_b = \frac{\sqrt{2}\, T_s\, \lambda^2}{\sqrt{\Delta\nu t C}\; \eta\, A_i\, \Omega_{syn}} \tag{17}$$

where

$$\begin{aligned}
\lambda &= 1.3\,\mathrm{mm} \\
A_i &= 2A_g = 2\times\pi/4\times(6.1)^2 = 58.44\,\mathrm{m}^2 \\
T_s &= 200K \\
\eta &= 0.75 \\
\Delta\nu &= 800\,\mathrm{MHz} \\
t &= 8\,\mathrm{hours} \\
C &= \text{number of baselines} = 25 \\
\Omega_{syn} &= \frac{\pi}{4}\theta^2 = \frac{\pi}{4}6''^{\,2} = \left(\frac{\pi}{4}\frac{6\times\pi}{3600\times 180}\right)^2 = 6.65\times 10^{-10} \qquad (18)
\end{aligned}$$

Substituting the above values we get

$$\Delta T_b = 0.68\mathrm{mK} \tag{19}$$

However we need a further factor of 2 in time as we have only one polarization receiver at each antenna and we need to switch the polarizations of the antennae. This would therefore require two tracks of 8 hours each day. The Millipol measurements of Clemens *et al.* (*ApJ*,**370**,257) obtained a total flux of around 30 Jy with percentage linear polarization around 4 %. The synthesized beam size was $30''$ (NRAO 12 m). We can convert this to T_b units.

$$T_b(\mathrm{meas}) = \frac{S}{\Omega}\frac{\lambda^2}{2k} = 35\,\mathrm{mK} \tag{20}$$

We thus obtain a signal-to-noise of about 50. It must be kept in mind that there are a number of factors that can complicate the above signal-to-noise ratio. These include uncertainity in the sytem temperatures, calibration and other instrumental effects. We have also assumed that the emission is diffused. If the emission were clumpy, then we would get a higher value for the SNR.