Problem 1: Consider, as we did in class, water moving in a tank. The left and right sides of the tank are located at $x = 0$ and $x = L$, respectively, and the tank is infinite in the $y$-direction. The depth of the water below its equilibrium level ($z = 0$) is $h$.

Assume now that the water motion is a *standing wave*. That is, the velocity potential $\phi$ is of the form

$$\phi(x, z, t) = \phi_0(z) f(x) \cos \omega t ,$$

and obeys the equation

$$\nabla^2 \phi = 0 .$$

(a) What are the boundary conditions to be imposed on $\phi(x, z, t)$?

(b) By applying all your boundary conditions, find expressions for $\phi_0(z)$ and $f(x)$.

(c) Find the period $P_0$ of the fundamental, or *sloshing*, mode.

(d) Estimate $P_0$ numerically for water in your bathtub. Is your answer reasonable?

Problem 2: In lecture, we solved the simple problem of an isothermal atmosphere in a uniform gravitational field $g$. Suppose, instead, that the atmosphere is isentropic, so that

$$P = K \rho^\gamma ,$$

where $K$ and $\gamma$ are constants.

(a) Derive an expression for the density $\rho$ as a function of height $h$. Your expression will contain, besides $\gamma$, $g$ and $h$, the molecular weight $\mu$, the gas constant $R$, and $T_0$ and $\rho_0$, the temperature and density, respectively, at ground level.

(b) Derive an expression for $dT/dh$. What is $dT/dh$ (in deg/km) for the Earth’s atmosphere?

(c) Do a little research to find out how your prediction compares with reality. What is the main factor accounting for the difference?

Problem 3: C & C, Problem 18

Problem 4: C & C, Problem 22