AY 202 Assignment 4

due: Tuesday, March 15

Problem 1: In class we considered an incompressible fluid of density $\rho$ that was initially stationary, but contained a line vortex of strength $\kappa$, lying along the $z$-axis. We found the evolution of the vorticity $\omega(R, t)$, where $R$ is the cylindrical radius. The vorticity diffuses outward, with a diffusion coefficient equal to the kinematic viscosity $\nu$.

(a) Now consider the evolving pressure within the fluid. First find analytically $p(R, 0)$, the initial pressure distribution. In your expression, let $p_0$ be the pressure at infinity. Sketch $p(R, 0)$.

(b) Next find $p(R, t)$. You should be able to express your answer in terms of the exponential function $E_1(x)$, where

$$E_1(x) \equiv \int_x^\infty \frac{\exp(-t)}{t} \, dt .$$

(c) Make several careful sketches of the pressure profiles for various times. If you are ambitious, you may look up tabulated values of $E_1(x)$ to help in your plots.

Problem 2: Shocks generate heat. Here you will determine the jump in $s$, the entropy per unit mass, across a viscous shock.

(a) Using the first law of thermodynamics, find an expression for $s$ of an ideal gas. Your expression should contain $C_v$ (the heat capacity per unit mass at fixed volume), $p$, $\rho$, and $\gamma$.

(b) Suppose gas with velocity $u_1$, density $\rho_1$, etc., crosses a stationary shock front, so that its velocity becomes $u_2$, etc. Write an expression for $s_2 - s_1$ in terms of the pre- and postshock pressures.

(c) Show that a “rarefaction shock,” i.e., one with $p_2 < p_1$, violates the second law of thermodynamics. That is, $s_2 - s_1$ would be negative under these circumstances.

Problem 3: C & C, Problem 27

Problem 4: C & C, Problem 33