

# Introduction to Millimeter Wavelength Interferometry

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1. INTRODUCTION

1.1. Radio Astronomy

- Radio astronomy is an observational science. We make images of the radio intensity $I(s, \nu, polarization, time)$

  Observations measure $I$ with sufficient resolution in $s, \nu, polarization, time$.
  Measure the source characteristics with sufficient sensitivity for the science goals.

- What we actually measure is

  $$I' = I * B + Noise$$

  where $B$ is the instrumental response, and $Noise$ is additive noise from the radio receivers and the atmosphere.

- Data reduction is the process of obtaining $I$ from our measurements, $I'$
  Must calibrate, and deconvolve the instrumental response to obtain $I$ from $I'$.

- Data Analysis is interpreting what $I$ means for astronomy.

- We need to know something about the sources of radio emission in order to design our telescopes and observations.

- Some of the 1st observations in a new waveband are usually surveys to find the distribution and nature of the sources.

  Later observations study the details of individual sources, or classes of sources.

- Instrumentation, and observing techniques together define a matched filter to possible observations to determine the characteristics of $I(s, \nu, poln, time)$.

- Source selection: astronomy, frequency, size, resolution, brightness, polarization.

  Some observations are a good match to the instrument.
  Others are more difficult: “impossible”, or “challenging”.
  We design new instruments to match new science goals.
  CARMA is a “new” instrument, evolving to meet new science goals.
1.2. Millimeter wavelength Radio Sources

- Astronomy from Comets to Cosmology
  
  Emission mechanisms: thermal and non-thermal

- Thermal emission is in quasi-equilibrium with the physical temperature.
  
  Black body - planets, asteroids, quiet sun
  Dust - grey body: dust emissivity at millimeter wavelengths
  Molecular lines: rotational-vibrational transitions
    
    Star formation regions, molecular clouds, stellar envelopes, YSO, evolved stars
    Galactic structure: CO and isotopes, CS, HCN, HCO+....
  
  Spiral and Dwarf galaxies: structure, gas content, rotation curves

- Non-Thermal emission is not in equilibrium with the physical temperature.
  
  Relativistic electrons & magnetic fields, synchrotron radiation, inverse compton, maser, cyclo-synchrotron
  
  SN remnants
  Radio galaxies: hot spots
  Active galaxies: nuclei
  Quasars, blazars, and Seyfert galaxies.
  Active sun: Solar flares.
  Masers: SiO, CH3OH
  
  Pulsars: steep spectrum, not important at millimeter wavelengths
  Comets: collisional (inner coma), and fluorescent excitation (Solar IR photons)

- Student Projects
  
  Is your project thermal or non-thermal emission?
  what category in above list?
1.3. Atmospheric windows

- Atmospheric windows at optical, IR, and radio frequencies

Earthbound astronomical observations are possible through atmospheric windows at optical, IR, and radio frequencies. (FIGURE 1 - Kraus)

- optical: 0.5 – 0.8 μm.
- IR: 1 – 1000 μm.
- Radio: 350 μm – 60 m.
- λ mm: 0.3 mm – 1 cm. (30 to 1000 GHz)

- λ mm astronomy atmospheric windows are defined by O₂ and H₂O lines into a number of bands bounded by tropospheric absorption at high freq and ionospheric absorption at low freq. (FIGURE 2 - Terrestrial Microwave window.).

- Away from the strong absorption lines, the atmospheric absorption is strongly dependent on atmospheric water vapor which has a scale height of about 2 km.

Observations at higher, drier sites have less absorption.

- CARMA site characteristics. (FIGURE 3 - Zenith atmospheric transmission).

CARMA altitude is 2196 m. Typical “good” conditions ~ 4 mm precipitable water vapor.

- Best winter weather. ~ 2 mm precipitable water vapor.
- c/f Mauna Kea at 4 km altitude often has less than 1 mm precipitable water vapor.

- CARMA observing bands.

CARMA currently has receivers for 2 bands.

- A 3 mm band, which can be tuned from ~ 75 – 115 GHz
- A 1 mm band, which can be tuned from ~ 210 – 270 GHz

In the 1 mm band the effects of atmospheric absorption are more serious and the advantages of a high, dry site are more obvious.
1.4. Intensity Units.

- Intensity units: $I(s, \nu, \text{polarization}, \text{time})$ Watts $m^{-2} \text{str}^{-1} \text{Hz}^{-1}$

- Brightness Temperature
  
  for black body radiation

  $$ I = \frac{2\nu^3}{c^2} \frac{1}{(e^{\nu/kT} - 1)} $$

  $T$ is the brightness temperature of an equivalent black body radiator.

  $$ h/k = 4.8 \ [GHz/100] \ K $$

- Rayleigh Jeans brightness temperature
  
  For $\nu/kT << 1$,

  $$ I = \frac{2kT}{\lambda^2} \left[ 1 - \frac{\nu}{2kT} + \ldots \right] $$

  Rayleigh Jeans brightness temperature, $I = 2kT_b/\lambda^2$

- Flux density

  $$ S = \int I \delta\Omega $$

  1 Jansky $= 10^{-26}$ Watts $m^{-2} \text{Hz}^{-1}$

  e.g. Planet with uniform brightness temperature $T$,

  $$ S = \frac{2kT}{\lambda^2} \Delta\Omega $$

  where $\Delta\Omega$ is the solid angle subtended by the planet.

- Brightness units: Kelvin, Jy/pixel, Jy/beam
1.5. Radio antennas: Collecting area and Aperture efficiency.

- **Collecting area** $\sim D^2$

  We must build antennas to collect radio photons from astronomical sources.

  Radio astronomy antennas are not used to transmit radio waves - as in radar astronomy - but it is often useful to think of antennas as transmitters.

  Amount of power we collect depends on the intensity, or brightness of radiation.

- **Antenna has an effective collecting area** $A(s)$; $s$ is direction in sky.

  For an elemental flat collector, the effective collecting area $= \delta A \times \cos(\theta)$

- **Power we collect**, 

  $$P = \int I(s) A(s) \delta \Omega \delta \nu \deltaWatts.$$  

- **Aperture efficiency**

  effective collecting area = aperture efficiency x collecting area

  aperture illumination, the weighted sum of electric fields across the aperture

  feed legs, subreflector blockage, spillover.

- **Surface accuracy**

  Aperture efficiency also depends on surface roughness.

  E-fields are summed in phase across the aperture.

  Ruze losses reduce effective collecting area by a factor $\exp[-(\frac{4\pi\sigma}{\lambda})^2]$

  Surface accuracy, $\sigma = \lambda/20$ corresponds to a Ruze loss factor 0.67.

- **CARMA antennas**

  Including thermal, pointing and focus errors, the overall RMS $\sim 75$ microns.

  at 3 mm wavelength, $\exp(-4 \pi 75/3000)^2 \sim 0.91$

  at 1.3 mm wavelength, $\exp(-4 \pi 75/1300)^2 \sim 0.59$

- **After correcting for pointing focus errors, we measure a surface RMS $\sim 30$ microns in ideal conditions. These are very good antennas at millimeter wavelengths.**
1.6. Radio antennas: Angular Resolution

• **Resolution** \( \sim \lambda/D \)

• Antenna Voltage pattern
  
  Antenna forms a weighted vector average of the E-field across the aperture
  
  \[ V(s) = \int W(r) \ E(r) \ \exp(2\pi i \ r.s/\lambda) \ \delta A \]

• Forward Gain of antenna add up the E-field across the aperture in phase in direction \( s_0 \).
  
  \[ \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \text{ in direction } s_0. \]

• In other directions a phase gradient across the aperture reduces the vector sum of the E-field across the aperture.

  **** DRAW HERE ****

  It is this phase gradient which gives an antenna resolution in direction.

  For a uniformly weighted rectangular aperture diameter \( D \),
  
  \[ V(s) = \int \exp(2\pi i x \sin(\theta)/\lambda) \ dx \quad [x = -D/2, D/2] \]
  
  \[ V(\theta) = \sin(\pi \theta \ D/\lambda)/(\pi \theta \ D/\lambda) \]

  For a uniformly weighted circular aperture diameter \( D \),
  
  \[ V(\theta) = J_1(\pi \theta \ D/\lambda)/(\pi \theta \ D/\lambda) \]

• Antenna power pattern
  
  \[ P(s) = V(s) \times V^*(s) \]

  Primary beamwidth, FWHM \( \sim 1.2 \ \lambda/D \)

• Interferometer power pattern
  
  Primary beam for an interferometer is the cross power pattern.

  For each antenna pair \((i, j)\),
  
  \[ P_{i,j}(s) = V_i(s) \times V_j^*(s) \]
1.7. Need for high resolution

- Early radio astronomy discovered radiation from the Galaxy, with unresolved maxima.
- Peaks named Cas A, Cygnus A, Sag A, Taurus A, Virgo A etc. Resolution $\sim 7$ degrees.
- Small optical telescope resolution, $\lambda/D \sim 5 \times 10^{-5}/10 \text{ cm} \sim 1$ arcsec.
- Arecibo, $\lambda/D \sim 6 \text{ cm}/300 \text{ m} \sim 36$ arcsec.
- Bonn, $\lambda/D \sim 1 \text{ cm}/100 \text{ m} \sim 20$ arcsec.
- IRAM 30m, $\lambda/D \sim 3 \text{ mm}/30 \text{ m} \sim 20$ arcsec.
- Confusion

  multiple sources in same beam.

- Small scale structure in radio sources
  - hot spots in radio galaxies (0.1 - 1 arcsec)
  - filaments in crab nebula (1 arcsec)
  - IR sources, OH and masers in star formation regions (1 - 10 arcsec)
  - molecular cloud structure (0.1 arcsec - several degrees)
  - spiral arm structure in galaxies (1 arcsec - a few degrees)
  - quasars components with $\lambda/D \sim 3 \text{ mm}/3000 \text{ km} \sim 0.2$ milliarcsec.
- Conclude that we need to observe structures over a wide range of angular scales.
  - single antennas can image structures larger than $\sim 20''$ at 3mm.
  - need effective antenna diameters $\sim 1$ kilometer to image arcsec structures.
- Techniques for high resolution observations
  - lunar occultations
  - interplanetary scintillations
  - interferometers
  - aperture synthesis
2. APERTURE SYNTHESIS

2.1. Aperture Arrays

- **Antenna size**: Area, Resolution, Confusion, Cost.

- An antenna provides:
  
  - Collecting area \( \sim \eta D^2 \)
  
  - Resolution FWHM \( \sim \lambda/D \).

- **Confusion**

  - Large antennas with enough sensitivity to detect more than one source within the beam are **confusion limited**.

  - We often have enough collecting area, but need more resolution.
  
  - Separate the functions of resolution and collecting area.
  
  - Build an array of smaller antennas.
  
  - Need to add up the E-field across this distributed aperture - preserving the relative phase of the wavefront across the array of antennas.

  - This is quite difficult. Not only do the electronics at each antenna need to preserve phase, but also atmospheric phase shifts distort the wavefront and we must compensate for these effects.

  - The problem is rather like adaptive optics.

  - We must keep the path lengths within \( \sim \lambda/20 \) to make an accurate telescope.

- **Surface accuracy** (FIGURE - SMA proposal 1984)

- **Cost of large antennas** \( \sim D^2 \lambda^{-0.7} \) (FIGURE)
2.2. Aperture Synthesis

- Consider aperture as set of antennas each with its own E-field and phase.

\[
\text{Voltage response} = \sum E_i \cos(\omega t + \phi_i)
\]

\[
\text{Power} = (\sum E_i \cos(\omega t + \phi_i))^2 = \sum E_i^2 + \sum E_i E_j \cos(\phi_i - \phi_j)
\]

- Sample different pieces of the aperture at different times.
  We must preserve relative phase across the whole aperture to synthesize a large telescope.

- Skeleton array, T-array contains all the relative spacings of square aperture.
  T-array has a different shaped beam because of different weights for each cross product, \(E_i E_j\)

- Minimum Redundancy Arrays.
  Linear arrays with few repeated spacings (Alan Moffet, 1968, IEEE AP-16, 172.):
  \[.1.2 .1.3.2 .1.3.2.2.1 .1.3.3.2 .1.5.3.2.2.\]
2.3. Earth Rotation Aperture Synthesis.

- An interferometer array of small antennas tracking a target across the sky samples different parts of the aperture plane due to the Earth’s rotation.

- Each antenna pair sweeps out a trajectory in the aperture plane.

- Response of a 2-element interferometer to point source:

\[
P(s) = V_i(s) \, V_j(s) \, \exp \left( \frac{2\pi i}{\lambda} \, \mathbf{b} \cdot \mathbf{s}_0 \right)
\]

where \( \mathbf{b} \) is the vector separation of the antenna pair \((i, j)\), and \( \mathbf{s}_0 \) is a unit vector in the direction of the source.

- coordinate systems: \( \mathbf{s}_0 = (b_N, b_E, b_Z) = (b_x, b_y, b_z) = (u, v, w) \)

\[
\begin{align*}
    u &= b_x \sin h + b_y \cos h \\
    v &= (-b_x \cos h + b_y \sin h) \sin \delta + b_z \cos \delta \\
    w &= (b_x \cos h - b_y \sin h) \cos \delta + b_z \sin \delta
\end{align*}
\]
2.4. Interferometer Observations

\[ V(t) = \int I(s)A(s - s') \exp \frac{2\pi i}{\lambda} b.s \, ds \]

\( s' \) is pointing center
\( s_0 \) is phase tracking center
\( \sigma = s - s_0 \) is the vector from the phase tracking center to the source

\[ V = \exp \frac{2\pi i}{\lambda} b.s_0(t) \int I(\sigma)A(s - s') \exp \frac{2\pi i}{\lambda} b.\sigma d\sigma \]

Instrumental terms  Source structure
\( \sigma = (x, y, z); \, b = (u, v, w) \)

- If \( \sigma \) is small, then we can neglect curvature of the sky and write this as a 2D Fourier transform. We must use a 3D transform, and other techniques to make images of large sources.

\[ V(u, v) = \int I(x, y)A(x - x', y - y') \exp \frac{2\pi i}{\lambda}(ux + vy) \, dx \, dy \]

- An interferometer array is a chromatic instrument, i.e. we must use a small range of \( \lambda \) or else we get bandwidth smearing. For wideband continuum sources, we can use bandwidth synthesis (mfs) to obtain more Fourier samples of the brightness distribution.

**********************************
DRAW 2-element interferometer
derive equation 1 for point source
then integrate over sky brightness
**********************************
2.5. Imaging

- We only have discrete samples of $V$, so we define a weighting function $W$, and evaluate:

$$I'(x, y) = \int W(u, v)V(u, v) \exp \left( -\frac{2\pi i}{\lambda} (ux + vy) \right) dx dy$$

- The weighting function $W$ can have any value where $V$ is sampled, and $W = 0$ where $V$ is not sampled. $I'$ is The Fourier transform of the product of $V$ and $W$, which is the convolution of the Fourier transforms of $V$ and $W$:

$$I'(x, y) = B(x, y) \ast [I(x, y)A(x, y)]$$

where

$$B(x, y) = \int W(u, v) \exp \left( -\frac{2\pi i}{\lambda} (ux + vy) \right) dudv$$

$B$ is the synthesized beam.

- For a point source of unit amplitude at the phase tracking center, $V(u,v)=(1,0)$. So the synthesized beam is the interferometer point source response, analogous to the point-spread function in an optical telescope.

- Choice of natural, uniform, robust weighting: effect on the synthesized beam

2.6. Deconvolution

- Deconvolve the synthesized beam response from the image $I'$.

$$I'(x, y) = B(x, y) \ast [I(x, y)A(x, y)] + \text{Noise}$$

CLEAN - iterative point source subtraction algorithm

MAXIMUM ENTROPY - maximum likelihood image consistent with the data.

- After deconvolution:

$I' = \text{Model}, M(x, y) + \text{Residual}, \text{Resid}(x, y)$ images

$M(x, y)$ is an estimate of $I(x, y)A(x, y)$

RESTOR - convolve Model by Gaussian with same FWHM as synthesized beam

$I_1(x, y) = G(x, y) \ast M(x, y) + \text{Resid}(x, y)$ – the default ”clean” image

$I_2(x, y) = G(x, y) \ast M(x, y)$

$I_3(x, y) = \text{Resid}(x, y)$
2.7. Properties of Synthesized Beam

- Resolution \( \sim \lambda/2D_{\text{max}} \), where \( D_{\text{max}} \) is the maximum antenna separation.
  Note factor of 2 since \( V(-u) = V^*(u) \)

- Aliasing of structures \( > \lambda/D_{\text{inc}} \), where \( D_{\text{inc}} \) is the sample interval in the \((u,v)\) data.

- Shortest spacing problem
  Source is “resolved out” if size \( > \lambda/2D_{\text{min}} \), where \( D_{\text{min}} \) is the shortest antenna separation.

- The shortest spacing is larger than the dish diameter \( D \), else collision or shadowing.

- Field of view of antennas \( \sim \lambda/D \).

- Each antenna configuration is sensitive to a range of angular sizes, \( \lambda/D_{\text{max}} - \lambda/D_{\text{min}} \)

- We must select sources with structures between \( \lambda/2D \) and \( \lambda/2D_{\text{max}} \) in order to image with a single pointing.

- Larger structures can be imaged using multiple pointing centers in a mosaic.

CARMA Synthesized beams and antenna spacings at 230 GHz.

<table>
<thead>
<tr>
<th>Array</th>
<th>( D_{\text{min}} ) [m]</th>
<th>( D_{\text{max}} ) [m]</th>
<th>Synthesized beam [( \theta )]</th>
<th>( \lambda/D_{\text{min}} ) [( \theta )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a-array</td>
<td>142</td>
<td>1884</td>
<td>0.13x0.11</td>
<td>2.8</td>
</tr>
<tr>
<td>b-array</td>
<td>82</td>
<td>946</td>
<td>0.33x0.26</td>
<td>4.7</td>
</tr>
<tr>
<td>c-array</td>
<td>25</td>
<td>373</td>
<td>0.8x0.6</td>
<td>15.6</td>
</tr>
<tr>
<td>d-array</td>
<td>10.2</td>
<td>148</td>
<td>1.7x1.4</td>
<td>39.2</td>
</tr>
<tr>
<td>e-array</td>
<td>7.5</td>
<td>66</td>
<td>3.5x2.8</td>
<td>52.3</td>
</tr>
</tbody>
</table>
2.8. Primary beam patterns

- Interferometer Primary beam pattern illuminates the Field of view.
- Primary beam for an interferometer is the cross power pattern.
- For each antenna pair \(((i, j),\)

\[ A_{ij}(s) = V_i(s) \times V_j^*(s) \]

- Illumination, or weighting, of the FOV is different for each antenna pair \(((i, j),\)

<table>
<thead>
<tr>
<th>Antennas</th>
<th>Equivalent diameter</th>
<th>FWHM</th>
<th>Nyquist interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.4 x 10.4</td>
<td>10.4</td>
<td>28</td>
<td>12.5</td>
</tr>
<tr>
<td>10.4 x 6.1</td>
<td>8.0</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>6.1 x 6.1</td>
<td>6.1</td>
<td>47</td>
<td>21.3</td>
</tr>
<tr>
<td>10.4 x 3.5</td>
<td>6.0</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>6.1 x 3.5</td>
<td>4.6</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>3.5 x 3.5</td>
<td>3.5</td>
<td>83</td>
<td>37.1</td>
</tr>
</tbody>
</table>
2.9. Holography

- Antenna Voltage pattern

  antenna forms a weighted vector average of the E-field across the aperture

  \[ V(s) = \int W(r) E(r) \exp(2\pi i r\cdot s/\lambda) \delta A \]

- We can measure the amplitude and phase of the voltage pattern by scanning one antenna across a strong radio source, or a radio transmitter, while using another antenna as a phase reference.

- The E-field distribution across the aperture is the Fourier transform of the Voltage pattern

  \[ W(r) E(r) = \int V(s) \exp(-2\pi i r\cdot s/\lambda) \delta s \]

- The measured aperture distribution gives the illumination - the weighting of the E-field - across the aperture. The phase gives the surface error.

  With sufficient resolution, we can clearly see the shadow of the feed legs and sub-reflector (FIGURE ).

  Phase gradient across the aperture is due to pointing error.

- After removing the phase gradient and a quadratic term due to focus error, we measure a surface RMS \( \sim 30 \) microns in ideal conditions. These are very good antennas at millimeter wavelengths.
3. OBSERVING PREPARATION

3.1. Overview of Observation Preparation

- Select Science target source(s)
  
  Science requirements: Resolution, Spatial dynamic range, Field of view, Sensitivity

  \[ \text{Source size} < \text{Primary beam FWHM}? \]

  True: Single pointing.
  False: Multiple pointings needed. Determine number of mosaic pointings.

  \[ \lambda/D_{\text{max}} > \text{Source size} > \lambda/D_{\text{min}}? \]

  True: Estimate image size and calculate Brightness sensitivity in synthesized beam.
  False: Not a good match to this antenna configuration.

- Select Observing frequency(s)

  \[ \text{Spectral line(s)}? \]

  True: Select correlator configuration. Calculate sensitivity at the velocity resolution needed.

- Select Calibration sources
  
  — close to target source.
  — strong enough to get a good calibration.
  — bandpass calibration.
  — absolute flux calibration.
  — Atmospheric phase coherence calibration.

- Prepare Observing Script.
3.2. Select Science target source(s)

- what are the Science Goals?
- what are the requirements?
- is this project a good match to CARMA capabilities?
- field of view? - angular size of source region.
  - source size? Single pointing center or Mosaic observation.
  - if multiple pointings needed are these contiguous?
- Angular Resolution required?
  - sufficient resolution to see the structures of interest.
- Sensitivity
  - what is the expected brightness of the source?
- Spatial dynamic range.
  - what is the range of angular scales we need to measure?
  - each antenna configuration is sensitive to a range of angular sizes, $\lambda/D_{max} - \lambda/D_{min}$

3.3. Select Observing frequency

- Spectral line observations?
  - select correlator configuration.
  - calculate sensitivity at the velocity resolution needed.
- Continuum observation.
  - maximum bandwidth currently three 500 MHz bands = 6 x 500 MHz windows.
  - calculate sensitivity.
- 3mm or 1mm observations?
  - what spectral lines are best?
  - what is the spectral index of the continuum emission?
  - calculate sensitivity.
3.4. Select Calibration sources

- Gain (amplitude and phase versus time calibration).
  observe a strong point source at 20 - 30 min intervals.
  usually a quasar within 20 degrees of target source.
  calibrator must be strong enough to get a good calibration.
  calculate sensitivity.

- Bandpass calibration.
  observe a strong calibrator to determine the instrumental frequency response
  calculate sensitivity.

- Absolute flux calibration.
  usually a planet whose flux can be calculated.

- Atmospheric phase coherence calibration.
  maybe needed if atmospheric turbulence is high.
  usually a weaker quasar close to the target source.
  check atmospheric phase monitor

- Calibration Errors
  for each calibration, the error in the measured visibility is given by:

  $\delta V' = \delta G \times V + \text{thermal noise}

  V is the true source visibility and $G$ is the calibration factor.
  prefer random thermal noise rather than systemic calibration errors.
  the calibrations are themselves limited by thermal noise.
  balance between time calibrating and time on target.
  for strong sources: must work harder on calibration.
  for weak sources: better to spend more time integrating on the target source.
3.5. CARMA Telescope Characteristics.

- CARMA is a heterogeneous array with six 10.4m and nine 6.1m antennas.
- Five array configurations giving resolution \(0.1, 0.3, 0.7, 1.8, 3.2\)" at 230 GHz.

3.6. Primary beam and Synthesized beam

- Primary beamwidth, \(FWHM \sim 1.2 \lambda/D\)
- Synthesized beamwidth, \(FWHM \sim \lambda/D_{max}\)
- Primary beam illuminates the sky brightness distribution.
  - sources smaller than primary beam can be observed with a single pointing
  - sources larger than the primary beams need multiple pointings
  - we MUST allow for the different primary beam patterns of 10 and 6m antennas.
- Spatial Dynamic Range
  - Each antenna configuration is sensitive to a range of angular sizes, \(\lambda/D_{max} - \lambda/D_{min}\)
  - Source structure larger than \(\lambda/D_{min}\) is "resolved out" by the interferometer.
  - Source structure larger than \(\lambda/D_{min}\) needs complementary single dish observations.

3.7. Mosaicing

- sources larger than the primary beam need multiple pointings
to image large scale structure needs overlapping pointings.
  - Nyquist sample interval \(\lambda/2D\) – see Table 1.
  - use same pointing centers for 10 and 6m antennas.
3.8. Sensitivity

- System Temperature

\[ T_{\text{sys}} = [T_{\text{sky}} (1 - e^{-\tau}) + T_{\text{source}} + T_{\text{ant}} + T_{\text{Rx}}] \times 2 e^\tau \]

(for single sideband observations with double sideband receivers)

Noise fluctuations \( \Delta T = T_{\text{sys}}/\sqrt{2Bt} \)

\( B \) is the bandwidth, \( t \) is the total integration time, and \( \tau \) is the atmospheric opacity.

Total integration time = N(N-1)/2 x time on source.

- Antenna Temperature

\[ k T_a = 0.5 \ S \ \eta \ A \]

- Antenna efficiency = illumination efficiency x surface efficiency

\[ \text{Jansky/Kelvin} = \frac{S}{T_a} = \frac{2k}{\eta A} \]

<table>
<thead>
<tr>
<th>antenna</th>
<th>2k/A</th>
<th>( \eta = 0.7 )</th>
<th>( \eta = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.4 m</td>
<td>32</td>
<td>46</td>
<td>65</td>
</tr>
<tr>
<td>6.1 m</td>
<td>94</td>
<td>135</td>
<td>189</td>
</tr>
</tbody>
</table>
• Flux sensitivity

\[ \Delta S = \Delta T \times Jy/K = \Delta T \left(2k/\eta A\right) \]

• Brightness Sensitivity

Flux density, \( S \) within the synthesized beam correspond to brightness temperature, \( T_b \).

\[ \Delta S = 2k \Delta T_b/\lambda^2 \times \delta \Omega \]

For a Gaussian synthesized beam \( \delta \Omega = \pi \theta_x \theta_y/4\log 2 \)

• Examples:

1. Continuum observations, BW=1500 MHz, DEC=30 degrees, time on source \( \sim 5 \) hr

2. Line observations, line width = 10 km/s, DEC=30 degrees, time on source \( \sim 5 \) hr

CARMA Sensitivity in \( \sim 5 \) hr

<table>
<thead>
<tr>
<th>GHz</th>
<th>Tsys</th>
<th>BW</th>
<th>( \eta )</th>
<th>( \Omega )</th>
<th>( \Delta S )</th>
<th>( \Delta T_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>200</td>
<td>1500 MHz</td>
<td>0.7</td>
<td>5.5 × 4.5″</td>
<td>0.4 mJy</td>
<td>1.9 mK</td>
</tr>
<tr>
<td>115</td>
<td>400</td>
<td>1500 MHz</td>
<td>0.7</td>
<td>4.8 × 3.9″</td>
<td>0.7 mJy</td>
<td>3.6 mK</td>
</tr>
<tr>
<td>230</td>
<td>500</td>
<td>1500 MHz</td>
<td>0.5</td>
<td>2.4 × 2.0″</td>
<td>0.9 mJy</td>
<td>4.5 mK</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>10 km/s</td>
<td>0.7</td>
<td>5.5 × 4.5″</td>
<td>8 mJy</td>
<td>40 mK</td>
</tr>
<tr>
<td>115</td>
<td>400</td>
<td>10 km/s</td>
<td>0.7</td>
<td>4.8 × 3.9″</td>
<td>15 mJy</td>
<td>72 mK</td>
</tr>
<tr>
<td>230</td>
<td>500</td>
<td>10 km/s</td>
<td>0.5</td>
<td>2.4 × 2.0″</td>
<td>13 mJy</td>
<td>63 mK</td>
</tr>
</tbody>
</table>

************ DEMO THE CARMA RMS CALCULATOR ************

http://cedarflat.mmarray.org/observing/tools/rms.html
4. OBSERVING PROCEDURES

4.1. Observer’s everyday responsibilities (John)

- creating and running the master schedule
- decide whether to do 1 mm or 3 mm
- data quality reports
- first line of troubleshooting

4.2. Troubleshooting (Dick, Marc)

- generator, air conditioning
- the anticollision system
- cryogenics
- rcvr tuning
- computer hangups
- clocks; resetting the time
- rebooting procedure:
4.3. Calibrating a new array configuration (John, Jin)

- changing IFLO connections in the pits
- running tilt, shimming the antennas
- entering new station coordinates
- finding pointing offsets.
- finding the delay centers
- TV and radio pointing
- finding a baseline
- entering new pointing offsets or baselines

CALIBRATION (Melvyn)

- Calibrations - gain, bandpass, polarization, pointing.
- Antenna based calibrations: amplitude and phase closure
- Atmospheric and instrumental phase characteristics
- Tsys and Jy/K
- Pointing
  - correlator calibration techniques.
  - Calibration interval.
    - interpolate the antenna gain amplitude and phase between observations of a known source.
- Observing scripts
  - selecting suitable observations for the target sources.
    - sensitivity
    - source size; mosaicing.

Table 2 lists the Nyquist sample interval for each antenna. Larger areas of sky may be imaged using a larger sample interval at the cost of reduced image fidelity and variation of the noise level across the image (see BIMA memo 73).
• CARMA correlator capabilities; selecting a correlator setup.
• choosing calibrators for gain, bandpass, flux and pointing.
• group discussion selecting student projects. (Douglas)
• Intro to preparing CARMA observing scripts (Marc)
• Student projects observed on CARMA array overnight.
5. DATA INSPECTION AND EDITING

5.1. Overview of data reduction procedure

- introduction to MIRIAD data reduction package.
- basic Miriad data format: header, history, uvdata, gains, bandpass
- inspecting uvdata: uvindex, uvlist, uvplt, uvspec
- selecting uvdata: keywords select= and line=
- flagging bad data with uvflag
- antenna based calibration; selfcal and mfcal. gpplt. gains bandpass polcal
- rewriting edited data sets with uvaver, uvcat, uvcal
6. CALIBRATION

The calibration of an interferometer array can be broken into 3 groups:

1) component values, e.g. cable lengths, which change only when the system is re-built.
2) array parameters which change after antenna moves.
3) instrumental parameters which change with time or ambient conditions.

(1) and (2) are handled by array staff and observers; (3) is responsibility of the user.

6.1. Calibration Requirements

(see BIMA memo 85)

1) IF fiber and cable lengths, IF delay centers.
2) Power levels, correlation coefficient corrections.
3) Receiver and phase lock tuning parameters.
4) Pointing parameters, receiver feed (horn) offsets, TV camera offsets
5) Primary beam patterns, focus, subreflector position, Holography
6) Antenna positions, antenna geometry, array geometry.
7) System temperature, opacity, atmospheric model.
8) Flux density scale, antenna Jy/K.
9) Polarization, leakage corrections.
10) Bandpass, IF and RF frequency response.
11) Gains: amplitude and phase versus time; frequency switching.
12. Atmospheric decorrelation, fast switching, and water line radiometry.
13) Single dish
14) Editing, flagging and blanking.
15) VLBI, phased arrays.
6.2. Calibration corrections to interferometer equation

\[ V(t) = G(t, f, p) \times \int I(s)A(s - s') \exp \frac{2\pi i}{\lambda} b.s ds \]

\( s' \) is pointing center
\( s_0 \) is phase tracking center
\( \sigma = s - s_0 \) is the vector from the phase tracking center to the source

\[ V = G(t, f, p) \times \exp \frac{2\pi i}{\lambda} [b.s_0(t) - delay0] \int I(\sigma)A(s - s' + \Delta s) \exp \frac{2\pi i}{\lambda} b.\sigma d\sigma + of f sets + noise \]

Instrumental terms Source structure

- \( G(t, f, p) \) complex gain versus time, bandpass, flux density and polarization calibration.
- \( delay0 \) instrumental and atmospheric delay calibration.
- \( A(s - s' + \Delta s) \) pointing, focus and primary beam calibration.
- phase switch by \( \pi \) to cancel offset.
- phase switch by \( \pi/2 \) to separate USB and LSB of LO1.
- observe calibration sources to estimate \( G(t, f, p) \)
  - how strong and how close to target source?

\[ V' = G \times V + thermal \ noise \]

for each calibration, the error in the measured visibility is given by:

\[ \delta V' = \delta G \times V + thermal \ noise \]

\( V \) is the true source visibility and \( G \) is the calibration factor.
prefer random thermal noise rather than systemic calibration errors.
the calibrations are themselves limited by thermal noise.
balance between time calibrating and time on target.
for strong sources: must work harder on calibration.
for weak sources: better to spend more time integrating on the target source.
6.3. Baseline-based calibration

\[ V'_{ij} = G_{ij}(t,f,p) \times V_{ij} + \text{noise} \]

Measured visibility = Gains \times True visibility + noise.

For a point source, \( V_{ij} = \text{Flux density (Jy)} \), so

\[ G_{ij}(t,f,p) = V'_{ij} + \text{noise} \]

6.4. Antenna-based calibration - selfcal

\[ G_{ij}(t,f,p) = g_i(t,f,p) \times g_j^*(t,f,p) \]

- Solve for \( g_i(t) \) as a function of time only.

\[ \chi^2 = \sum (V'_{ij} - g_i g_j^* V_{ij})^2 / \sigma^2_{ij} \]

summed over time interval, \( \Delta t \). Can include frequency channels and pointings.

6.5. Atmospheric phase calibration

Fast switching and/or paired antenna calibration.

Effective distance between source and calibrator

\[ d = \sqrt{[h(s - s')]^2 + [v \Delta t]^2} \sim 140m \]

For \( h \sim 1 \text{ km height of atmospheric fluctuations (maybe multiple), } h \sim 1 \text{ km} \)  
source-calibrator angle, \( s - s' \sim 5 \text{ deg, turbulence speed, } v \sim 10 \text{ km/s, and } \Delta t \sim 10s. \)  
\( h(s - s') \sim 100m, \text{ and } v \Delta t \sim 100m \)

Reduced atmospheric phase fluctuations on antenna baselines > 140m
6.6. **Bandpass calibration - mfcal**

\[ G_{ij} (t, f, p) = g_i (t, f, p) \times g_j^* (t, f, p) \]
\[ g_i (t, f, p) = g_i (t) \times g_i (f) \]

- Solve for antenna gains, \( g_i (t) \) as a function of time.
- Solve for bandpass, \( g_i (f) \), as a function of frequency.
- Solve for XX and YY, total intensity polarizations.

\[ \chi^2 = \Sigma (V'_{ij} - g_i g_j^* V_{ij})^2 / \sigma_{ij}^2 \]

6.7. **Polarization calibration - gpcal**

\[ G_{ij} (t, f, p) = g_i (t, f, p) \times g_j^* (t, f, p) \]
\[ g_i (t, p) = g_i (t) \times g_i (p) \]

\[ \chi^2 = \Sigma (V'_{ij} - g_i g_j^* V_{ij})^2 / \sigma_{ij}^2 \]

- Solve for antenna gains as a function of time, \( g_i (t) \).
- Solve for instrumental polarization, \( g_i (p) \).
- Solve for polarization leakage, and XY phase.
- Solve for source polarization if and YY total intensity polarizations.
6.8. Self calibration

- Solve for antenna gains as a function of time using a model image.

\[ G_{ij} (t, f, p) = g_i (t, f, p) \times g_j^*(t, f, p) \]
\[ \chi^2 = \sum (V'_{ij} - g_i g_j^* V_{ij})^2 / \sigma_{ij}^2 \]

- Same algorithm as antenna-based calibration for point source.
- Use model image for true visibility, \( V_{ij} \) to solve for antenna gains, \( g_i \).
- Make new image using these gains, deconvolve, and make new model.
- Clip model image to exclude noise.
- Iterate to find model and gains consistent with measured visibilities and noise \( \sigma_{ij} \).
6.9. Calibration Errors

- For each calibration, the error in the measured visibility is given by:

\[ \delta V' = \delta G \times V + \text{thermal noise} \]

where \( V \) is the true source visibility and \( G \) is the calibration factor.

- Prefer random thermal noise rather than systemic calibration errors.

- The calibrations are themselves limited by thermal noise.

- Balance between time calibrating and time on target.

For strong sources: must work harder on calibration.

For weak sources: better to spend more time integrating on the target source.

- Sensitivity for each baseline is given by:

\[ \delta S = Jy\text{per}k \times T_{sys}/\sqrt{2B \Delta t} \]

Table 1 gives the sensitivity per baseline for each combination of antennas assuming 80% aperture efficiency, \( T_{sys}=200 \) K, and 1 minute integration. The table lists the sensitivity for calibrating antenna gains (amplitude and phase) in a 4 GHz bandwidth, the bandpass with a 1 MHz spectral resolution, and the brightness sensitivity at 5" angular resolution. For antenna based calibration, the RMS is reduced by \( \sim \sqrt{(N_{ants})} \)

<table>
<thead>
<tr>
<th>Antennas</th>
<th>Equivalent diameter</th>
<th>JyperK</th>
<th>RMS (4 GHz)</th>
<th>RMS in 1 MHz</th>
<th>RMS in 5’’</th>
</tr>
</thead>
<tbody>
<tr>
<td>m x m</td>
<td>m</td>
<td>Jy/K</td>
<td>[mJy]</td>
<td>[Jy]</td>
<td>[K]</td>
</tr>
<tr>
<td>10.4 x 10.4</td>
<td>10.4</td>
<td>41</td>
<td>13</td>
<td>0.8</td>
<td>4</td>
</tr>
<tr>
<td>10.4 x 6.1</td>
<td>8.0</td>
<td>69</td>
<td>23</td>
<td>1.4</td>
<td>7</td>
</tr>
<tr>
<td>6.1 x 6.1</td>
<td>6.1</td>
<td>118</td>
<td>39</td>
<td>2.4</td>
<td>12</td>
</tr>
<tr>
<td>10.4 x 3.5</td>
<td>6.0</td>
<td>122</td>
<td>40</td>
<td>2.5</td>
<td>12</td>
</tr>
<tr>
<td>6.1 x 3.5</td>
<td>4.6</td>
<td>208</td>
<td>68</td>
<td>4.3</td>
<td>21</td>
</tr>
<tr>
<td>3.5 x 3.5</td>
<td>3.5</td>
<td>359</td>
<td>118</td>
<td>7.4</td>
<td>36</td>
</tr>
</tbody>
</table>
7. IMAGING

7.1. Imaging extended sources

- Review of basic math
- Brightness distribution is FT of sampled visibility data.
- We only have discrete samples of $V$, so we define a weighting function $W$, and evaluate:

\[ I_0(x,y) = \int W(u,v)V(u,v) \exp\left(\frac{-2\pi i}{\lambda}(ux + vy)\right) \, dx \, dy \]

- The weighting function $W$ can have any value where $V$ is sampled, and $W = 0$ where $V$ is not sampled. $I'$ is The Fourier transform of the product of $V$ and $W$, which is the convolution of the Fourier transforms of $V$ and $W$:

\[ I'(x,y) = B(x,y) \ast [I(x,y)A(x,y)] \]

where

\[ B(x,y) = \int W(u,v) \exp\left(\frac{-2\pi i}{\lambda}(ux + vy)\right) \, du \, dv \]

$B$ is the synthesized beam.

7.2. Gridded Fast Fourier Transform

- In order to use a Fast Fourier Transform algorithm, we must resample the $(u,v)$ points onto a gridding $u$-$v$ plane. The data are multiplied by the weighting function $W$, convolved by a function $C$, and resampled onto a regular grid by $\Pi$

\[ [(V \times W) \ast C] \times \Pi \leq FFT \Rightarrow [(I \ast B) \times c] \ast \Pi \]

- The Fourier transform of the gridded $u$-$v$ data is an image of the sky brightness distribution $I$, convolved the synthesized beam $B$, multiplied by $c$, and convolved by $\Pi$.
- The convolution by $\Pi$ replicates the image at intervals $1/\delta uv$, where $\delta uv$ is the sample interval of the gridded $u$-$v$ data.
- Aliasing in the sky brightness image is minimized by choosing a function $C$, so that its Fourier transform $c$ falls to a small value at the edge of the image.
- The width of the image is $1/\delta uv$. 
7.3. Image size and pixel sizes in sky plane and (u,v) plane

- The image size and sample interval in the gridded (u,v) and the (x,y) sky plane are related through the Fast Fourier Transform.

\[
\text{Image size} = N \ \delta_{xy} = 1/\delta_{uv} \\
(u,v) \text{ plane size} = N \ \delta_{uv} = 1/\delta_{xy}
\]

- Choosing the pixel and image size.
  
  The image size, \( N \) and sample interval \( \delta_{xy} \) are chosen in sky plane.
  
  \( x \) and \( y \) coordinates may be independently specified.
  
  The image size must be large enough to contain the FOV being imaged.

\[
N \ \delta_{xy} > \text{field of view}
\]

- The (u,v) plane must be large enough to accommodate the sampled (u,v) data.

\[
N \ \delta_{uv} > 2 \times D_{\text{max}}/\lambda
\]

- MIRIAD \textbf{invert}
  
  default \( \delta_{xy} \approx \lambda/3D_{\text{max}} \approx 1.5 \times \text{Nyquist} 
  
  default \( N \ \delta_{xy} = \text{Primary beam FWHM} 

- Mosaicing \textbf{invert options=mosaic}
  
  image size, \( N \), specifies subimage for each pointing
  
  image size, \( N \ \delta_{xy} \) = full width at 5\% in primary beam (\texttt{pbplot})
  
  smooth overlap between subimages for each pointing in mosaic image.
  
  imsize should not be a power of 2.
7.4. Spectral Line Imaging

- Use same beam for all frequency channels.
- Spectral line images are scaled by $\nu/\nu_0$.
- Spectral channels can be averaged without interpolation.
- Use same beam for deconvolution.

$$I'(x, y, \nu) = B(x, y, \nu_0) \ast [I(x, y, \nu)A(x, y, \nu_0)]$$

7.5. Bandwidth smearing and MFS imaging

- Wideband image is smeared in radial direction

$$\Delta r/r = \Delta \nu/\nu$$

- Multifrequency Imaging
  use spectral channels to make wideband image.
  each frequency channel is independent uv sample.

7.6. Deconvolution

- Fourier Transform is a linear operation. Deconvolution is not.
- CLEAN
  Hogbom, Clark, SDI, and MX algorithms
  iterative source subtraction from specified region of image
  removes sidelobes which are outside the region cleaned.
  converges when residual image is consistent with noise.
- RESTOR
  convolve the subtracted source model by Gaussian beam
  add to the residual image
· MAXEN

Maximum Entropy Deconvolution

Maximize entropy, $H$, consistent with observations:

$$H = -\Sigma \left(I \log(I/M)\right)$$

subject to constraint

$$\chi^2 = \Sigma \left(\frac{(V' - V)^2}{\sigma^2_{ij}}\right)$$

Entropy measure: either "gull" (-$p \log(p/e)$) or "cornwell" (-$\log(\cosh(p)$)

$I$ is the MEM image, $M$ is a default image.

Convergence and behavior of MAXEN depends strongly on RMS.

Initial image model and total FLUX constraint also possible.

Gradient search algorithm:

- primary beam weight the current MEM image
- FFT to get predicted visibility $V'$ from image and beam .
- accumulate $\chi^2$
- generate new MEM image.
- iterate until MEM image is consistent with the data.

· MIRIAD clean, maxen, uvmodelex handle spectral line image models

- Hybrids, a-priori models, uniqueness

  CLEAN is good for point sources.
  MEM is good for smooth, extended brightness distributions.
  MEM and CLEAN image models in units Jy/pixel
  UVMODEL subtracts image models from the uv-data.
  Image model is consistent with the data. It is not unique.
7.7. Self Calibration

- Uses an image model to calibrate the uv-data.
  
  Solve for $g_i(t)$ as a function of time.
  
  $$\chi^2 = \sum (V_{ij}^r - g_i g_j^* V_{ij})^2 / \sigma_{ij}^2$$
  
- summed over time interval, $\Delta t$.
  
- MIRIAD selfcal handles spectral line image models:
  
  - multiple frequency channels
  
  - multiple pointing centers
  
  - point source and planet models

7.8. Large Field Imaging

- arrays are good at mapping compact structures.
  
- single dishes are good at mapping large scale structure.
  
- telescopes are matched filters to a set of spatial scales.
  
- importance for getting correct answers for spectral index, etc;
  
- many astrophysical quantities are derived from ratios
  
  $$^{13}CO/^{12}CO = I(^{13}CO) * B(^{13}CO) / I(^{12}CO) * B(^{12}CO)$$
  
  $$SI = log[I(\nu_1) * B(\nu_1)] - log[I(\nu_2) * B(\nu_2)] / [log(\nu_1) - log(\nu_2)]$$
  
  WRONG answer, since $a/b$ is not equal to $(a-a')/(b-b')$
  
  $a'$ and $b'$ are the missing large scale structure not sampled by an interferometer array
7.9. Short Spacing Problem

- interferometer array samples spacings from $D_{\text{min}}$ to $D_{\text{max}}$.
  - source structure with angular scale $\lambda/2D_{\text{min}}$ is not sampled.
  - shortest spacing $> \text{dish diameter } D$, else collision or shadowing.
- importance for getting correct answers for spectral index, etc;
- negative sidelobes due to extended structure
- filling in missing spacings with larger single dish or Ekers-Rots scheme
- An interferometer gives some partially sampled shortest interferometer spacings, then none at all less than some $uv_{\text{min}}$.
  - uv-samples in annulus between $uv_{\text{min}}$ and $uv_{\text{max}}$
  - synthesized beam is a $J_1(x)/x$ for $uv_{\text{max}}$, minus a $J_1(x)/x$ for $uv_{\text{min}}$.
  - This is the negative basin around the source.
- shortest interferometer spacings, are a just few Fourier components, i.e. sine waves, with quite high amplitudes.
  - Images may look better without them.
  - convolve the uv-plane inner edge to attenuate the few short spacings sampled by interferometer.
7.10. Sampling Short Spacings

- single dish observations
  - single dish samples spacings from 0 to $D_{ant}$.
  - interferometer array samples spacings from $D_{min}$ to $D_{max}$.
  - ideal single dish $D_{ant} \sim 2 \times D_{min}$
  - need overlap for calibration.

- multiple beams for each antenna
  - (Welch 1984, Granada IAU/URSI conference proceedings)
  - illumination pattern rotates on sky for Alt-Az antennas.
  - cross-talk between beams gives variable DC offsets.

- Heterogeneous antenna arrays.
  - CARMA 10.4, 6.1 and 3.5m antennas.
  - original MMA (ALMA) proposal was for 4m and 10m antennas.

- Mosaicing
  - Ekers and Rots 1979, Groningen IAU
  - Cornwell non-linear mosaicing using MEM
  - Clean mosaic.
7.11. Single Dish Observations

- Single dish beam $A(x, y)$ convolves the sky brightness distribution $I(x, y)$
  \[
  I'(x, y) = I(x, y) \ast A(x, y) \times \Pi(x, y)
  \]

- Fourier transform gives the visibility data sampled in the uv-plane
  \[
  V'(u, v) = V(u, v) \times a(u, v) \star \pi(u, v)
  \]

  $a(u, v)$ is the weighting of spatial frequencies sampled by the single dish.

  sampled visibilities are convolved by $\pi$, the Fourier transform of $\Pi$.

  uv-data are aliased if $\pi < 2D_{ant}/\lambda$

  sample single dish observations with $\Pi < \lambda/2D_{ant}$ to avoid aliasing.

7.12. Combining Single Dish and Interferometer Observations

- single dish samples spacings from 0 to $D_{ant}$.
  zero spacing gives the total flux in the image

- interferometer array samples spacings from $D_{min}$ to $D_{max}$.

- ideal single dish $D_{ant} \sim 2 \times D_{min}$
  need overlap for calibration.

- IMMERGE linearly merges single dish and interferometer images.
  deconvolve single dish observations. Need single dish beam.
  multiply by interferometer image primary beam pattern.
  sample overlapping uv-data in an annulus.
  establish consistent calibration between single dish and interferometer.
  merge the two images together

- MOSAIC joint deconvolution of a mosaic and single dish image.

- MOSAIC using the single dish image as a default image.
7.13. Ekers and Rots

- Mosaic observations measure a range of visibilities around each (u,v) point.
  - Two antennas, diameter D, separated by d, sample the wavefront between d-D and d+D.
- Measure visibility at pointing center \( s' \)
  \[
  V(u, s') = \int I(s)A(s - s') \exp \frac{2\pi i}{\lambda} usds
  \]
- Phase gradient across aperture correlates different spacings on antenna surface.
- Observing a grid of pointings measures different Fourier samples of wavefront.
- Fourier transform w.r.t. pointing measures visibilities around each (u,v) point.
  \[
  V(u, u') = a(u') \times \int I(s)A(s - s') \exp \frac{2\pi i}{\lambda} (u + u')sd\]
  \[
  V(u, u') = a(u') \times V(u + u')
  \]
- In practice we recover \(~ D/2\lambda ~) around each (u,v) sample.
  - i.e. uv-tracks are \(~ D/\lambda ~) wide, multiplied by \( a(u') \)
- Need single dish observations to sample \(<\sim D/2\lambda ~) with good SNR.
8. MOSAIC IMAGES

8.1. Astronomical Requirements

- Millimeter wavelength Imaging from Comets to Cosmology
  - Hale Bopp 1 AU 3 arcmin 0.1 arcsec (75 km)
  - Orion Core 500 pc 40 arcsec 0.1 arcsec (50 AU)
  - Cas A 3 kpc 5 arcmin 1 arcsec (3000 AU)
  - IC342 nucleus 2 Mpc 1 arcmin 1 arcsec (10 pc)
  - Cygnus A 200 Mpc 3 arcmin 1 arcsec (1 kpc)
  - Cluster Core 2 Gpc 3 arcmin 10 arcsec (100 kpc)

- Field of View > Primary beam width (~ 1 arcmin at λ 3mm)
- Resolution < ~ 1 arcsec.
- Spatial dynamic range ~ 1000.

8.2. Heterogeneous arrays

- CARMA primary beams from 6.1 x 6.1, 10.4 x 10.4, and 6.1 x 10.4
- ALL CARMA images are mosaics – even with only 1 pointing center.
- primary beam patterns depends on illumination of antennas.
- mosaicing algorithms clip primary beam at ~ 5% level,
  avoids uncertainties in primary beam at low levels.
  Within 5% level, primary beam pattern ~ Gaussian.
8.3. Mosaicing Algorithms

- Two problems:
  1. Mapping a large Field of view.
  2. Imaging large scale structures.

8.4. Mapping Field of view larger than primary beam

- same as single dish mapping
- scan antennas across the source and sample sky at Nyquist
- Primary beams $A(x, y)$ convolve the sky brightness distribution $I(x, y)$
  sampled at intervals $\delta \theta$ by $\Pi$
  \[
  I'(x, y) = I(x, y) \ast A(x, y) \times \Pi(x, y)
  \]
- Fourier transform gives the visibility data sampled in the uv-plane
  \[
  V'(u, v) = V(u, v) \ast a(u, v) \ast \pi(u, v)
  \]
  $a(u, v)$ is the weighting of spatial frequencies sampled by the primary beams.
  sampled visibilities are convolved by $\pi$, the Fourier transform of $\Pi$.
  uv-data are aliased if $\pi < 2D_{ant}/\lambda$
  sample antenna pointings with $\delta \theta < \lambda/2D_{ant}$ to avoid aliasing uv-data.
8.5. Imaging large scale structures

- Interferometer data image $I'(x, y)$

$$I'(x, y) = \int W(u, v)V(u, v) \exp\left(\frac{-2\pi i}{\lambda}(ux + vy)\right) dxdy$$

$$I'(x, y) = I(x, y) \times A(x, y) \star B(x, y)$$

- in Fourier plane the measured visibility $V'(u, v)$ is:

$$V'(u, v) = V(u, v) \star a(u, v) \times W(u, v)$$

$W(u, v)$ is the sampling in the uv-plane.

$a(u, v)$ is the weighting of spatial frequencies sampled by the antennas.

image is convolved by $B(x, y)$.

image is aliased if $\delta uv > D_{ant}/2\lambda$.

sample uv-data $\delta uv < D_{ant}/2\lambda$ to avoid aliasing in sky plane.
8.6. Linear Mosaicing Algorithms

- linear mosaic is primary beam weighted linear combination of pointings.

\[ I'(x) = g(x) \sum (A(x-x_i) I_i(x)/\sigma_i^2)/(\sum A^2(x-x_i)/\sigma_i^2) \]

- MIRIAD invert options=mosaic – just like single field imaging.

- \( g(x) \) is residual primary beam pattern to keep noise constant across mosaic image

- synthesized beam is different for each pointing.

  synthesized beam has \( Npts \) image planes.

- sidelobes \( \sim 1/Npts \)

- pointing errors may be common to adjacent pointings

- gain and phase errors may be common to adjacent pointings

- observing strategy is to sample all pointings in short time interval.

- selfcal can be used with multiple pointing centers.
8.7. Maximum Entropy Mosaic Deconvolution

- Maximize entropy, \( H \), consistent with observations:

\[
H = -\sum I \log(I / M e)
\]

subject to \( \chi^2 \) constraint summed over all pointings \((x', y')\)

\[
\chi^2 = \sum (V'(u, v, x', y') - V(u, v, x', y'))^2 / \sigma(u, v, x', y')^2
\]

- Entropy measure: either "gull" (-p log(p/e)) or "cornwell" (-log(cosh(p))

- \( I \) is the MEM image, \( M \) is a default image.

- can do joint deconvolution of a mosaic and single dish image.

- can use single dish image as a default image.

- convergence and behavior of MOSMEM depends strongly on RMS.

- initial image model and total FLUX constraint also possible.

- gradient search algorithm:
  - primary beam weight the current MEM image
  - FFT to get predicted visibility \( V' \) from image and beam.
  - accumulate \( \chi^2 \) over all pointings \((x', y')\)
  - generate new MEM image.
  - iterate until MEM image is consistent with the data.

- Maximum Entropy deconvolution pioneered by Tim Cornwell
  find image which is consistent with all the data.
  may be many such images, so chose the most likely one.
  Maximum Entropy image subject to \( \chi^2 \) constraints.
  gradient search algorithm in MEM and \( \chi^2 \) image.

- MIRIAD uses an efficient user-friendly algorithm pioneered by Bob Sault.

- MIRIAD mosmem and mossdi are like MAXEN and CLEAN for single field deconvolution.

- both algorithms start with images, not uv-data.
8.8. Sampling Requirements

- Sampling rates are set by both 10m and 6m antennas.
- Nyquist sample interval for uv data, $\delta uv = D_{ant}/2\lambda$.
- Nyquist sample interval for pointings, $\delta \theta = \lambda/2D_{ant}$.
- The number of pointings, $Npts \sim \Omega/(\delta \theta)^2$.
- Nyquist sample rate $\sim baseline/\lambda \times (2D_{10m}/\lambda)^2 \times 2\lambda/D_{6m} \times \Omega \times sdot$,
  
  $D_{ant}$ is antenna diameter, $\Omega$, source size, and $sdot = 2\pi/24/3600$.
  
  sample rate $= \sim (100m/\lambda) \times 2\lambda/6m \times Npts \times 7.2710^{-5}$
  
  Nyquist sample interval $\sim 400/Npts$ seconds

- The uv data oversampled by the larger antenna,
- pointing is oversampled by the smaller antenna.
- No loss in sensitivity; the oversampled data are accounted for.
- 10m antennas on long baselines for better SNR and reduce uv data sample rate.
- sample rate $= baseline/\lambda \times 2\lambda/D_{ant} \times \Omega/(\lambda/2D_{ant})^2 \times sdot$

  Larger antennas must sample faster.

  On-the-fly mosaicing needed for ALMA.
8.9. Image Fidelity as a Function of Source Size

- CARMA array can image ~ 32" diameter with a single pointing center using the 15-antenna D-configuration at 230 GHz. must treat as mosaic observation.

- up to ~ 64" diameter can be imaged with a 7-pointing hexagonal mosaic using the 15-antenna D-configuration at 230 GHz.

- image fidelity decreases as the source size increases.

- joint deconvolution gives the best image fidelity.

- image fidelity very dependent on high quality single dish data.

- default image for lower quality single dish data.

- fidelity improved by mosaic with 23-antenna DZ configuration.
  - short spacings with 3.5m antennas sample large scale structure.
  - and cross calibrate single dish and interferometer observations.

- CARMA memo 38
8.10. **Mosaicing Sensitivity**

- **Single pointing**
  \[ \Omega_{\text{source}} < \Omega_{\text{beam}} \]
  
  **Flux sensitivity**
  
  For \( N \) antenna array, total observing time = \( N(N - 1) \times t \)
  
  \[ \Delta S = 2k/\eta A \times T_{\text{sys}}/\sqrt{(2Bt)} \]
  
  \[ \Delta S \sim 2kT_{\text{sys}}/\sqrt{(2Bt)} \times (1/\eta ND_{\text{ant}}^2) \sim 1/ND_{\text{ant}}^2 \]

- **Brightness sensitivity**

  \[ S = 2k T_b/\lambda^2 \times \Omega_{\text{synth}} \]
  
  \[ \Delta T_b = T_{\text{sys}}/\sqrt{(2Bt)} \times (1/\eta ND_{\text{ant}}^2) \times \lambda^2/\Omega_{\text{synth}} \]

  Brightness temperature sensitivity is scaled up by beam filling factor:

  \[ \Omega_{\text{synth}} \sim (\lambda/D_{\text{max}})^2 \text{ and } \Omega_{\text{beam}} \sim (\lambda/D_{\text{ant}})^2 \]

  \[ \Delta T_b = T_{\text{sys}}/\sqrt{(2Bt)} \times 1/\eta N \times (D_{\text{max}}/D_{\text{ant}})^2 \]
  
  \[ \Delta T_b = T_{\text{sys}}/\sqrt{(2Bt)} \times 1/\eta N \times (\Omega_{\text{beam}}/\Omega_{\text{synth}}) \]

- **Mosaic Observations**

  \[ \Omega_{\text{source}} > \Omega_{\text{beam}} \]

  number of pointings = \( \Omega_{\text{source}}/\Omega_{\text{beam}} \)

  observing time per pointing = \( t \times (\Omega_{\text{source}}/\Omega_{\text{beam}}) \)

  **Flux sensitivity**

  \[ \Delta S = 2k/\eta A \times T_{\text{sys}}/\sqrt{(2Bt)} \times (\Omega_{\text{source}}/\Omega_{\text{beam}})^{1/2} \]

  \[ \Omega_{\text{beam}} \sim (\lambda/D_{\text{ant}})^2; \ A \sim ND_{\text{ant}}^2 \]

  \[ \Delta S \sim 2kT_{\text{sys}}/\sqrt{(2Bt)} \times (\Omega_{\text{source}}^{1/2}/\eta N \lambda D_{\text{ant}}) \sim 1/ND_{\text{ant}} \]

  **Brightness sensitivity**

  \[ S = 2k T_b/\lambda^2 \times \Omega_{\text{synth}} \]

  \[ \Delta T_b = T_{\text{sys}}/\sqrt{(2Bt)} \times 1/\eta ND_{\text{ant}} \times \lambda/\Omega_{\text{source}}^{1/2} \]
8.11. Hexagonal pointing grids

- Pointing grids must sample the source.
- Easy to build custom grids using hexagonal patterns.
- Hexagonal grid sampled at Nyquist spacing $\lambda/2D_{\text{ant}}$
  - Oversampling by $\sqrt{3}/2$ helps the mosaicing process.
  - Less time per pointing, but no loss in sensitivity.

- MIRIAD tasks: \texttt{hex}, \texttt{hexc}

8.12. Primary beam and Image sensitivity

- \texttt{IMLIST options=}\texttt{mosaic} List the mosaic table for an image.
- \texttt{MOSSEN} determines the rms and gain of a mosaiced image. Both of these parameters are a function of position (and also mildly frequency dependent).
  
  To avoid noise amplification at the edge of mosaiced regions, Miriad does not normally totally correct for the primary beam beyond a certain point.

- \texttt{MOSPSF} determines the point spread function for a linear mosaic produced by \texttt{invert options=}\texttt{mosaic}.
  
  Strictly speaking, the PSF varies with position and frequency. However if the pointing grid is fairly complete and the individual synthesized beam patterns are similar, the PSF is reasonably independent of position. It is also usually a good approximation that it is independent of frequency.

- \texttt{DEMOS} multiplies model of the sky primary beam patterns at a number of different pointing centers. It produces a different output for each pointing center. Thus this task performs the inverse of mosaicing. The input pointing centers and the primary beam size are indirectly specified by a visibility dataset.

Because the outputs of DEMOS have primary beams applied, they can be used for comparison with visibility data and uncorrected images. In particular \texttt{SELFICAL} cannot handle a model which is primary beam corrected, though it can handle a visibility data file containing multiple pointings. Thus you could use DEMOS to break the model into several models which are not primary beam corrected.
9. DATA ANALYSIS

9.1. uv-data and images are complementary.

- uv-data are the observational data but hard to interpret
  uv-data can be compared with model brightness distributions.
- images are derived data products but easier to interpret

9.2. Fitting uv-data.

- UVFIT for point, disk, gaussian, shell or ring models
  needs initial estimates of source parameters.
- UVAMP plot uv-data in annuli versus uvdist.

9.3. Fitting images

- IMPLOT, CGDISP are the primary plotting tasks for images.
- ELLINT integrates an image in elliptical annuli.
- IMFIT fits point, disk, gaussian or beam objects to an image.
  needs initial estimates of source parameters.
- VELPLOT analyse spectra, position-velocity, and integrated velocity maps.

9.4. Comparing uv-data with images

- UVMODEL compares uv-data with an image model.
  the model can add, subtract, multiple, divide or replace the uv-data.
  can also process mosaic and polarization data.
9.5. Position Fitting

- use uvfit or imfit

- interferometer phase gives the position. Phase ~ transit time.

- measured position is w.r.t. reference frame of calibration sources.
  use a set of reference sources around target source.

- fringe phase $2\pi$ ambiguity eliminated by multiple antenna separations.

- accuracy $\sim 1/SNR \times \lambda/D_{max}$, where $SNR$ is the signal-to-noise.

  interferometer phase $= \frac{2\pi}{\lambda} b.s$

  $b$ is antenna baseline. $s$ is the source position

  measured phase $= \frac{2\pi}{\lambda} (b.\Delta s + \Delta b.(s - s_0))$

  $\Delta s$ is the estimated position offset of the target.

  $\Delta b$ is the error in the antenna separation.

  $s - s_0$ is the angle between the target and weighted mean for reference objects.
10. CARMA HARDWARE - I. Receivers and Calibration (Dick)

- system block diagram; receiver, cal load, local osc, phaselocks, fiber, downconverter, correlator

- energy collected if observing 20 Jy source for 1 yr; would need to observe for 100,000 years to heat 1 drop of water by 1 C

- receiver types:
  - bolometers: not suitable for interferometry because they don’t preserve phase
  - HEMT amplifiers: not yet competitive at 1mm
  - heterodyne rcvr: downconvert to lower freq in a nonlinear device
  - SIS mixers: photon-assisted tunneling; not a Josephson effect
  - cryogenics; closed-cycle refrigerators, compressors
  - local oscillator: Gunn oscillator

- must be synchronized between all antennas; discuss in lecture 2

- both USB and LSB are downconverted to IF; can be separated with 90 degree phase switch; also defer to lecture 2

- combining LO and signal: mylar beamsplitter

- receiver and system temperature

- calibration:

- ideally, calibrate on loads outside the earth’s atmosphere

- the chopper wheel method

- CARMA sensitivity calculator
11. INTRODUCTION TO CARMA SOFTWARE SYSTEM (Marc)

- logging in; directory structure
- overview of computers, control and systems, data flow, etc
- CARMA Programs
- Observer tools
12. CARMA HARDWARE -II - local oscillators, phaselocks (Dick)

- review system block diagram, heterodyne system, local oscillator
  - independent oscillators, 100 GHz, synchronized to fraction of one cycle over periods of hours (sounds hard)
  - basic phaselock: mix with reference, low pass filter, generate correction voltage; keeps phase relationship fixed
  - CARMA phaselock chain; synth, YIG, Gunn, 10 MHz, 50 MHz
  - numerical example: synth = xxx, YIG = yyy, LO = zzz
  - fiber system; linelength correction
  - lobe rotation
  - compute differential doppler shift due to earth’s rotation for 100 GHz signal incident on 2 antennas 10-m apart: 0.24 Hz
  - lobe rotators
  - interferometer response for a double sideband conversion system
  - need to offset freq of 1st LO as well as insert delays; can be understood as removing differential doppler shift due to earth’s rotation
  - phaselocks; the LO system
  - cable length measurement system
  - phase switching; Walsh functions
  - sideband separation by phase switching; note that only signals common to an antenna pair can be separated; noise appears in both sidebands
  - fiber optic hardware
  - converting to flux density; aperture efficiency; source flux table
13. CARMA SOFTWARE SYSTEM II (Marc)

- What software does an observatory need?
- Timekeeping
- Antenna and device control
- Monitor system (a.k.a. telemetry)
- Interprocess communication
- Data pipelines
- User interface
- Software engineering standards
14. CARMA HARDWARE - III - correlator (Dick)

- review system block diagram
  - Antennas, beam patterns, etc. (James)
  - Correlator (DaveW)
- correlator is detector and spectrometer for the array
- XF vs FX
- delays, 2nd LO lobe rotation, sideband separation
- correlator modes
- FPGA’s
- noise source
- basic architecture of computer control system, CAN nodes
15. INTERPRETING ERRORS IN IMAGES (Douglas)
16. INTRODUCTION TO SINGLE DISH OBSERVING (Marc)

- Autocorrelation
- Position switching
- Computing the source spectrum
- Baseline (continuum) subtraction
- Efficiency improvements
17. CARMA FUTURE PLANS

- polarization measurements (Melvyn)
  - interferometer response LR, RL, etc in terms of Stokes parameters
  - Walsh function polarization switching schemes
  - instrumental leakage terms and how to solve for them
  - mapping procedures
- Atmospheric phase fluctuations and what we plan to do about them
  - like floppy backup structure on a big telescope
  - causes decorrelation, ruins aperture efficiency
  - show results at long and short baselines
  - phase structure function
- Paired antenna calibration system using 3.5m antennas.
- Rapid switching; put calibrator in grid file;
  - observe weak nearby calibrator often, strong faraway calibrator less often
- calibrating by observing the total power; need for extreme gain stability; typical results
- A-configuration
- Carlstrom’s 1 cm system
- wideband receivers
- 23-antenna CARMA array
Fig. 1.— CARMA site opacity and rms path for 2008 4-hour time periods
Fig. 2.— CARMA site 3mm observing thresholds versus opacity and rms path for 2008 4-hour time periods.
Fig. 3.— CARMA site 1mm observing thresholds versus opacity and rms path for 2008 4-hour time periods
Fig. 4.— Saturn image simulations using CARMA C configuration and CZ configuration.