Theory of Interstellar Phases

1. Relevant Observations
2. Linear Stability Theory
3. FGH Model
4. Update and Summary

References

Tielens, Secs. 8.1-5

Field ApJ 142 531 1965 (basic stability theory)

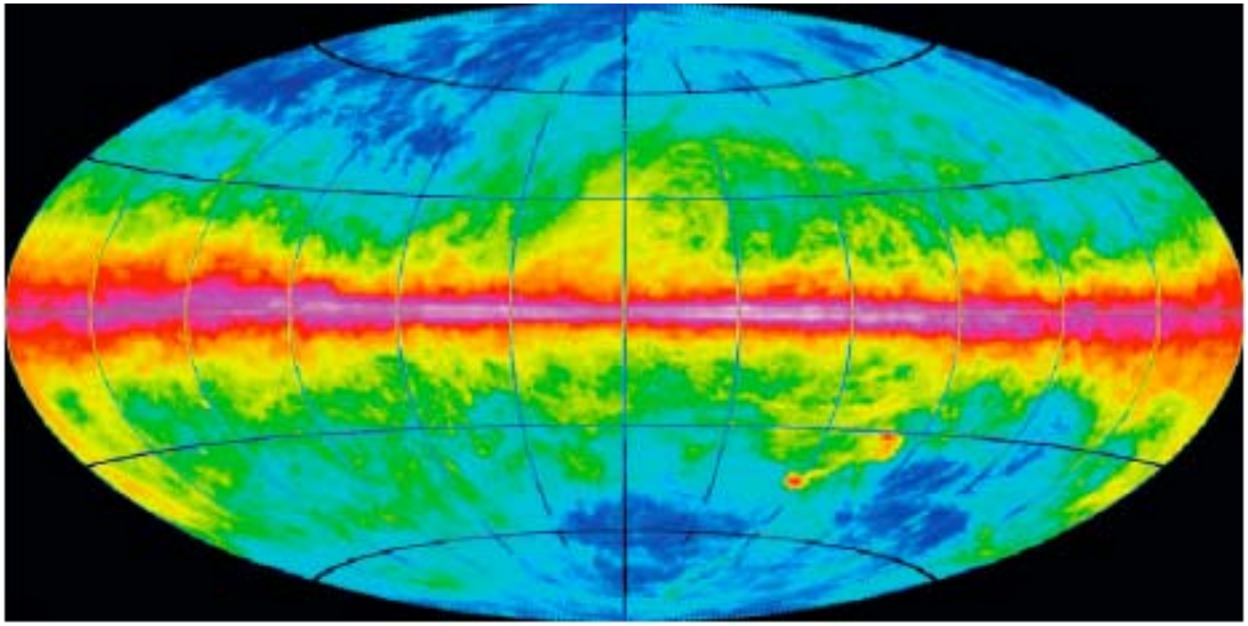
Field, Goldsmith & Habing, ApJ 155 L149 1969 (two-phase model)

Draine ApJS 36 595 1978 (photoelectric heating)

Shull & Woods ApJ 288 50 1985 (X-rays)

Wolfire et al. ApJ 443 152 1995, ApJ 587 278 2003 (updated models)

1.Observations of Phases



Integrated 21-cm Emission of the Milky Way
Leiden/Argentina/Bonn Merged Catalog
Velocity range form -450 to +400 km/s

Kalberla et al. A&A 440 775 2005

The velocity profiles yield the physical properties of the component phases (Lec10).

Summary of CNM & WNM Heiles & Troland II)

1. Emission is ubiquitous - absorption is not
2. CNM & WNM are observationally distinct
3. WNM is 60% of total HI
4. Median columns and

   CNM 0.5×10^{20} cm^{-2}

   WNM 3×10^{20} cm^{-2}

5. CNM median (column density weighted) temperature is ~ 70 K

5. WNM temperature distribution is broad (500 20,000 K) with a peak near 8,000 K, but ~ 50% to WNM is thermally unstable (below).

7. CNM clouds are *sheet like*

Tentative Physical Properties of Phases

Phase	Observations	*T*	*n*
CNM	HI etc. abs. lines	75	40
WNM	HI emission lines	4,000	0.075
WIM	DM, EM, H*α* em.	0.0375	8,000
HIM	UV abs., soft X-rays	0.003	10^6

- The numbers are only meant to be suggestive.
- Poorly known filling factors have been ignored.
- *Assumed* pressure equilibrium at nT = 3,000 cm^{-3} K.
- Heiles & Troland find 50% of the WNM to be in the thermally unstable range from ~ 500-5,000 K.

Thermodynamic Phases?

Since before 1965, observations suggest two types of HI:

Low density (WNM) - dominant at high temperature

High density (CNM - dominant at low temperature

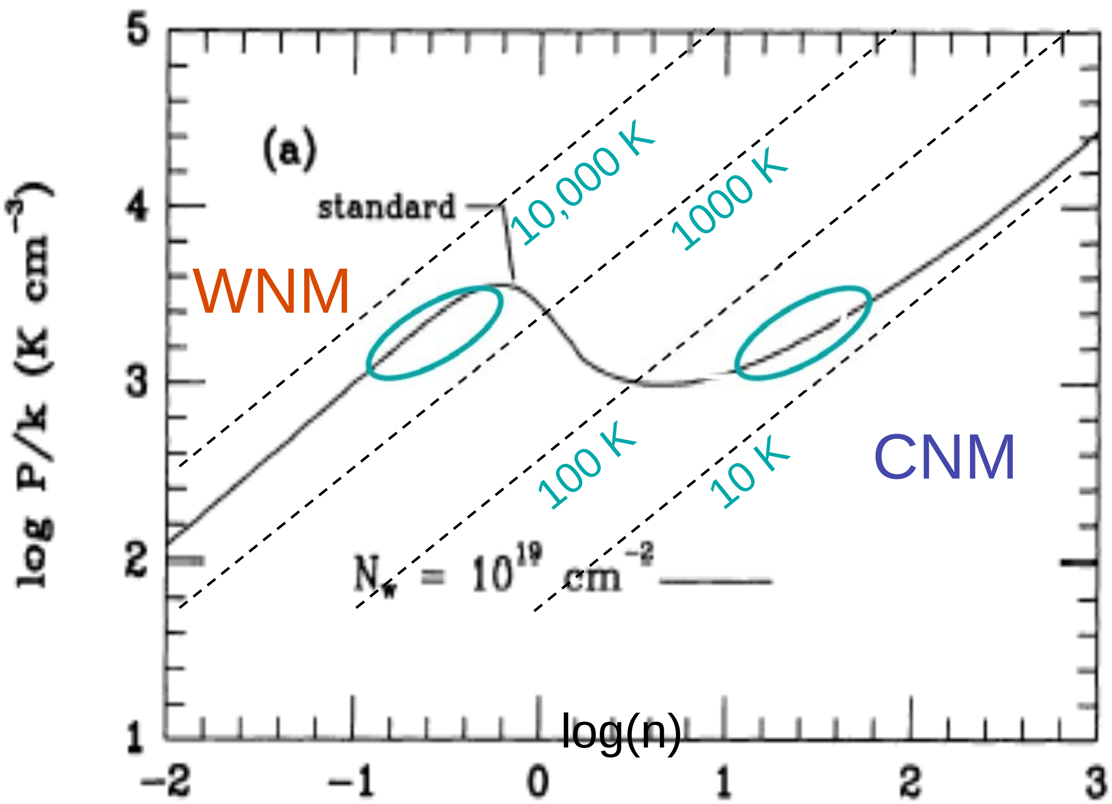
consistent with thermal balance considerations

But are they to be considered as thermodynamic phases familiar from terrestrial physics and chemistry?

One motivating analogy is terrestrial water, where phase equilibrium is observed near sea level. On this basis one might expect HI to condense into (and evaporate from) the CNM in response to external conditions – a kind of *interstellar weather*. But we know that, although water vapor plays a key role, meteorology is more than just phase diagrams – but of course we can still try.

HI Phase Diagram

Phase diagram
(solid curve)
near the Sun
Wolfire et al. 1995



Coexistence for
$P/k \sim 10^3 - 10^4$ cm^{-3}K

T(WNM) ~ 8,000 K
T(CNM) ≥ 50 K

2. Linear Stability Theory

(Field ApJ 142, 531,1965)

Recall from Lec04 the equations for a ***single*** fluid

$$\frac{d\rho}{dt} + \rho \nabla \cdot \vec{v} = 0$$

$$\rho \frac{d\vec{v}}{dt} + \nabla p = 0$$

$$\rho(T \frac{ds}{dt} + \Lambda_{\rm net}) = \nabla \cdot (\kappa \nabla T), \qquad \Lambda_{\rm net} = \rho^{-1}(\Lambda - \Gamma)$$

where $\Lambda_{\rm net}$ is the ***net heating rate for unit mass***, and the entropy change for a polytropic gas is

$$\rho T ds = \frac{1}{\gamma - 1}(dp - \gamma \frac{p}{\rho} d\rho)$$

Ionization, gravity, and magnetic fields are ignored.

Field's Analysis

As in the elementary theory of sound propagation, the hydro equations are linearized (around a steady state):

$$\rho = \rho_0 + \delta\rho,\ \ v = \vec{v}_0 + \delta\vec{v},\ \ T = T_0 + \delta T$$

maintaining the standard thermal balance condition

$$\Lambda_{\rm net}(\rho_0, T_0) = 0$$

When *δρ, δv, and δT* vary as exp[*i(kz - ωt*]*)*, i.e., as waves traveling in the direction z, a cubic ***characteristic equation*** is obtained for *ω.* The condition (on Λ_{net}) for it to have roots corresponding to growing modes gives a ***thermal instability condition*** (*thermal* because it is a condition on the cooling function Λ_{net}).

Heuristic Derivation

Rather than going through the algebra of the Field's theory as just outlined, we apply a simplified analysis directly on the heat equation. Consider the time evolution (*dt*) of a small fluctuation (δT, $\delta \rho$, etc.) for a co-moving fluid element:

$$\delta \Lambda_{\mathrm{m}} = \left(\frac{\partial \Lambda_{\mathrm{m}}}{\partial s} \right)_A \delta s$$

With one thermodynamic variable *A* fixed (e.g., the pressure)

$$T d(\delta s) = -\delta \Lambda_{\mathrm{m}} dt$$

this equation becomes

$$\frac{1}{\delta s}\left(\frac{d}{dt}\delta s\right) = -\frac{1}{T}\left(\frac{\partial \Lambda_m}{\partial s}\right)_A \equiv \frac{1}{\tau_{gr}}$$

where τ_{gr} is a characteristic growth time.

Heuristic Derivation (cont'd)

From the last equation we conclude that

$$\left(\frac{\partial \Lambda_{\mathrm{m}}}{\partial s}\right)_A < 0$$

is the condition for exponential growth (or ***thermal instability***), and that τ_{gr} is the ***growth rate of the instability.*** With the aid of thermodynamics, the derivative can be expressed in terms of T and ρ (e.g., using $Tds_\rho = c_\rho dt$ *and* $Tds_p = c_p dt$ etc.,), we can get these more specific conditions:

$$\left(\frac{\partial \Lambda_{\mathrm{m}}}{\partial T}\right)_\rho < 0 \qquad \text{isochoric}$$

$$\left(\frac{\partial \Lambda_{\mathrm{m}}}{\partial T}\right)_p = \left(\frac{\partial \Lambda_m}{\partial T}\right)_\rho - \frac{\rho}{T}\left(\frac{\partial \Lambda_{\mathrm{m}}}{\partial T}\right)_T < 0 \qquad \text{isobaric}$$

FGH Model of Two Phases in Equilibrium

ApJ 155 L149 1969

Basic Idea: Gas in thermal balance (*Λ*=*Γ*) can coexist at the same *p* with two (*n,T*) combinations, conceived as ***cool clouds embedded in a warm intercloud medium*****,** now referred to as CNM and WNM.

Thermal Balance:

$$\Lambda_m = \rho^{-1}(\Lambda - \Gamma),$$

$$\Lambda = \lambda n_{\mathrm{H}}{}^2 \quad \text{and} \quad \Gamma = \gamma n_{\mathrm{H}},$$

$$\Lambda = \Gamma \quad \Rightarrow \quad n_{\mathrm{H}} = \frac{\gamma}{\lambda}.$$

λ is sensitive and γ insensitive to T

$\Lambda \sim n_H{}^2$: cooling comes mainly from low-density sub-thermal collisions)

$\Gamma \sim n_H$:heating comes from an external source, e.g radiation, cosmic rays, turbulence, shocks,

FGH

Thermal balance is described by equations which express the density or the pressure as a function of *T* :

$$n_{\mathrm{H}} = \frac{\gamma}{\lambda} \quad \text{or} \quad \frac{p}{\gamma} = \left(\frac{n}{n_{\mathrm{H}}}\right) \frac{kT}{\lambda} \quad \text{using} \quad p = nkT,$$

NB: n/n_{H} is the number of particles for H nucleus, e.g. for an atomic region with few electrons, $n/n_{\mathrm{H}} \cong 1.1$

FGH used cosmic ray heating and CII fine-structure heating:

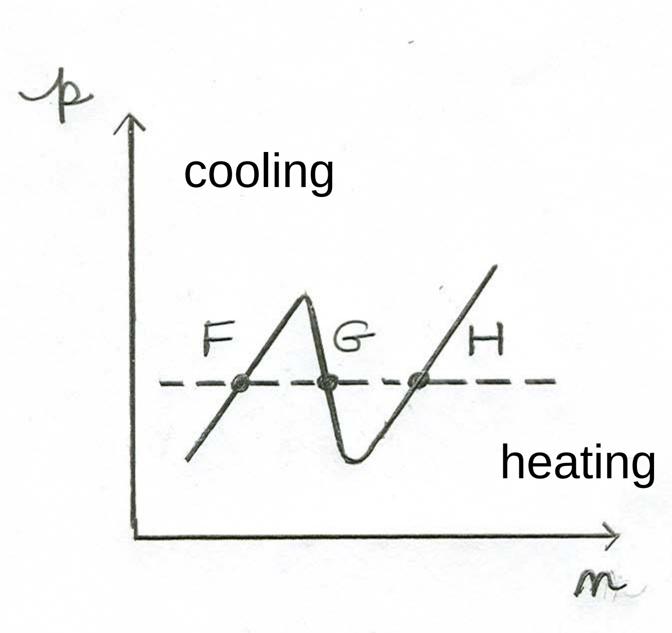
$$\gamma_{\mathrm{CR}} = \varsigma\varepsilon_{\mathrm{CR}} \quad \text{with} \quad \varsigma \approx 10^{-15}\,\mathrm{s}^{-1} \quad \text{and} \quad \varepsilon_{\mathrm{CR}} \approx 20\,\mathrm{eV}$$

$$\lambda = 2x_{\mathrm{C}} C_{ul} e^{-91\mathrm{K}/T} \, (k_{\mathrm{B}} 91\mathrm{K})$$

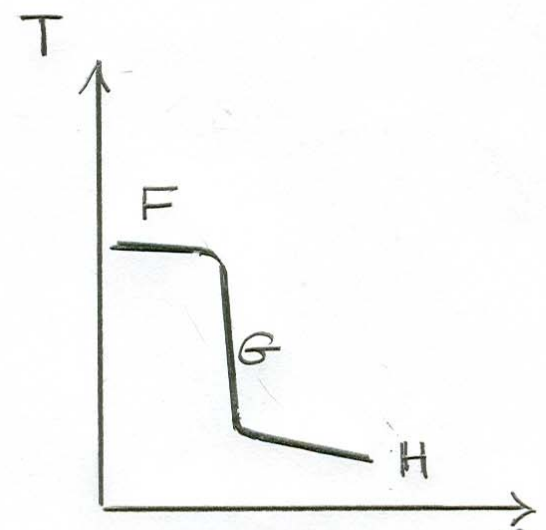
C_{lu} is the abundance weighted sum of e and H collisional de--excitation rate coefficients.

FGH

F = stable WNM G = unstable H = CNM



p vs.1/v
backwards van der Waals

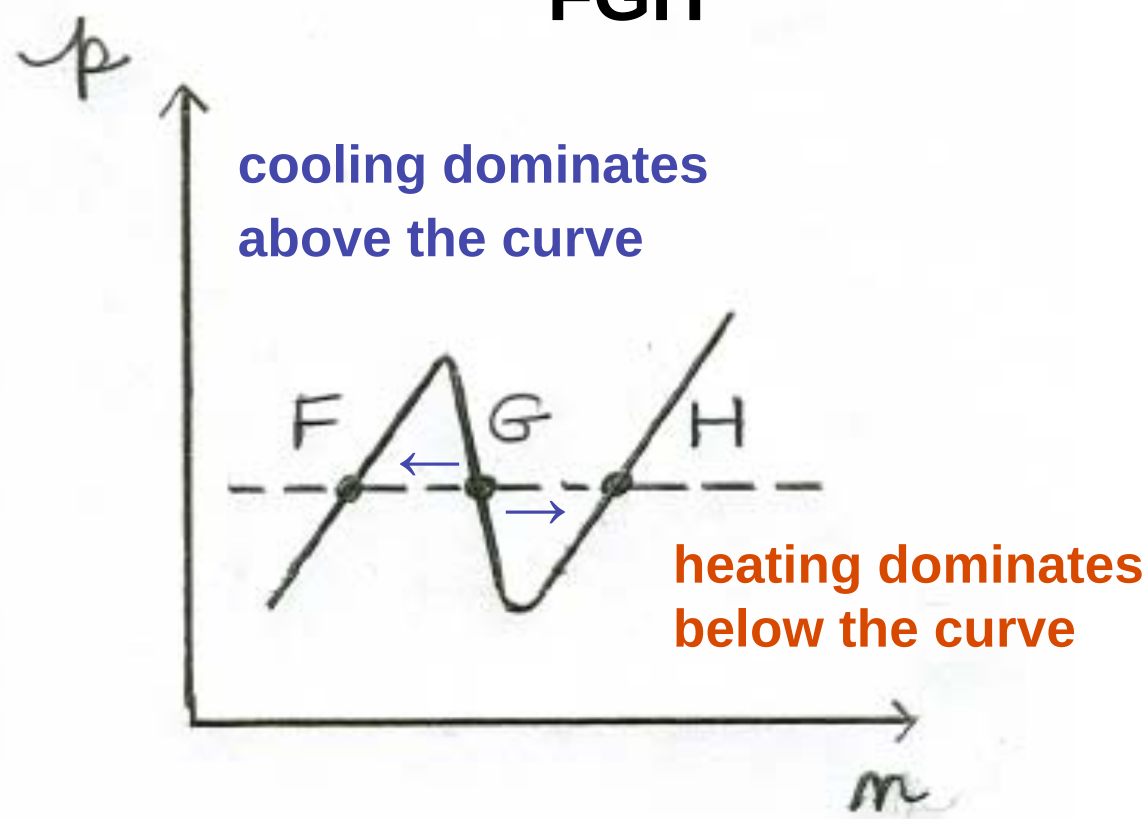


Equivalent Version

$$\frac{\partial}{\partial n}(nkT) < 0 \quad \Rightarrow \quad \frac{\partial T}{\partial n} < -\frac{T}{n}$$

Positive compressibility is the thermodynamic *stability* condition .

FGH



Displacing G at constant *p* makes it go to F or H, whereas similar perturbations restore F & H (at constant *p*, *T* and *n* are anti-correlated): G is unstable to isobaric perturbations, but F and H can be in stable pressure equilibrium.

Timescales for Growth of Instabilities

Earlier we found that the *growth time* for thermal instability is

$$\frac{1}{\tau_{gr}} = -\left(\frac{\partial \Lambda_m}{\partial s}\right)_A = -\frac{1}{c_A}\left(\frac{\partial \Lambda_m}{\partial T}\right)_A$$

The usual heat capacities per unit mass are

$$c_\rho = \frac{3}{2}\frac{k}{m} \quad \text{and} \quad c_p = \frac{5}{2}\frac{k}{m}, \quad m = \rho/n$$

where $m = \rho/n$ is the mass per particle (*not* equal to the mass per H nucleus, $m' = \rho/n_{\mathrm{H}} = 1.35$). In this notation, the cooling function per unit mass is

$$\Lambda_m = \frac{1}{m'}\left(n_{\mathrm{H}}\lambda - \gamma\right)$$

Growth Timescale

It is now straightforward to evaluate the growth times and instability conditions for the case of constant γ and λ (a function of T only) using $p = (n/n_{\rm H})\, n_{\rm H} k_{\rm B} T$

$$\frac{1}{\tau_\rho} = \frac{m}{m'} \; \frac{n_{\rm H}}{{}^{3}\!/_{2}\, k} \left(-\frac{d\lambda}{dT} \right) \qquad \frac{1}{\tau_p} = \frac{m}{m'} \; \frac{n_{\rm H}}{{}^{5}\!/_{2}\, k} \left(-\frac{d\lambda}{dT} + \frac{\lambda}{T} \right)$$

Isochoric fluctuations are unstable for λ decreasing with T.

Isobaric fluctuations are unstable for λ either (i) decreasing with T or (ii) increasing less rapidly than λ/T.
Equivalently, the isobaric condition is:

$$\frac{d \ln \lambda}{d \ln T} < 1$$

Cooling and Growth Times

The thermal instability growth times are closely related to the more familiar *cooling time*:

$$\frac{1}{\tau_{\rm th}} = \frac{\Lambda}{\frac{3}{2} nkT} = \frac{m}{m'} \frac{n_{\rm H}}{\frac{3}{2} k} \frac{\lambda}{T}$$

This provides a lower limit to the isobaric growth time.

We estimate it for the unstable state (G in FGH), using the results of Wolfire et al. (1995) to be discussed below:

$$T = 2000\ {\rm K} \qquad n_{\rm H} = 1.5\ {\rm cm}^{-3} \qquad x_{\rm e} = 0.005$$

$$\tau_{\rm th} = 0.94\ {\rm Myr} \qquad \tau_{\rm rec} = 3.4\ {\rm Myr}$$

These timescales are not short compared to dynamical time scale in the ISM.

Recombination Time Scale

The recombination timescale is

$$\tau_{\rm rec} = \frac{1}{n_{\rm e}\alpha(T)} = \frac{1}{n_{\rm e}\ 2.24\text{x}10^{-10}\, T^{-0.725}\,\text{cm}^3\text{s}^{-1}} = \frac{141\ \text{yr}\ T^{0.725}}{n_{\rm e}\text{cm}^3}$$

Estimates -

CNM: $n_{\rm e}$ = 0.01,T= 80 K ---> 3x10^5 yr
WNM: $n_{\rm e}$ = 0.10,T= 8,000 K ---> 1x10^6 yr

The recombination time scale is long, often longer than than the thermal time scale

Chemical (ionization) equilibrium is an important art of the overall thermal balance, e.g., it enters into the cooling function. Similarly, The ionization and heating are usually produced by the same external agent. The phase diagram (*p* vs *n*) should probably be better represented as surfaces (*p* vs $n, x_{\rm e}$).

Dynamical Timescale

A typical timescale is the sound crossing time, a measure of time for pressure equilibration.

$$\tau_{\text{dy}} = \frac{L}{c_{\text{s}}} \qquad c_{\text{s}} = \sqrt{\frac{kT}{m}} = 0.145 \text{ km s}^{-1}\sqrt{T}$$

$$\tau_{\text{dy}} = 6.74 \text{ Myr} \left(L/\text{pc}\right) T^{-1/2}$$

We then estimate:

$$\tau_{\text{dy}} = 674{,}000 \text{ yr} \left(L/\text{pc}\right) \qquad \text{for the CNM } (T \approx 100\text{K})$$

$$\tau_{\text{dy}} = 67{,}400 \text{ yr} \left(L/\text{pc}\right) \qquad \text{for the WNM } (T \approx 10{,}000\text{K})$$

Bottom Line: Dynamical and recombination time scales ~ Myr are usually longer than thermal time scales , especially in the case of the WNM

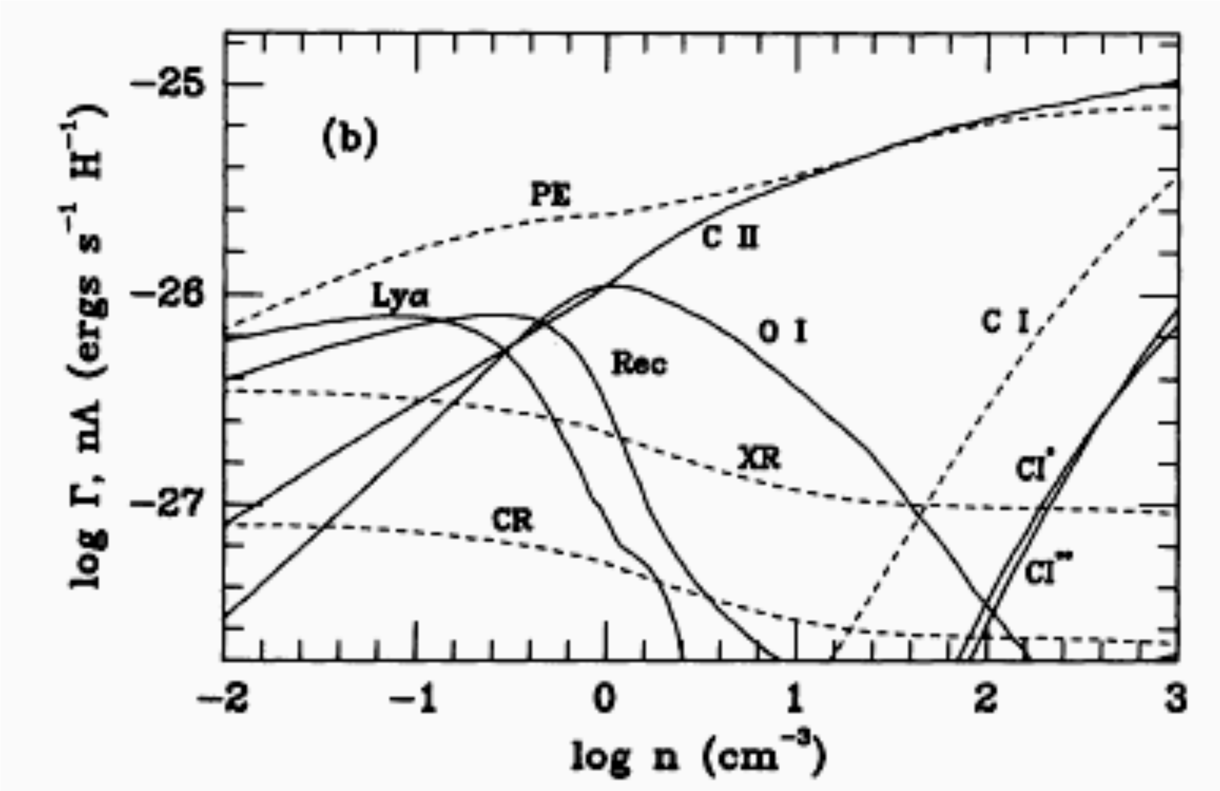
Updated FGH Models

"The Neutral Atomic Phases of the ISM"
Wolfire, Hollenbach, McKee, Tielens
ApJ 443 152 1995 (with Bakes) & ApJ 587 278 2003

Steady state thermal *and* chemical balance for the 10 most abundant elements plus grains and PAHs, subject to:

process	ionization	heating	cooling
FUV	♪	♪	
CRs & X-rays	♪	♪	
recombination	♪		♪
line emission			♪

Heating and Cooling



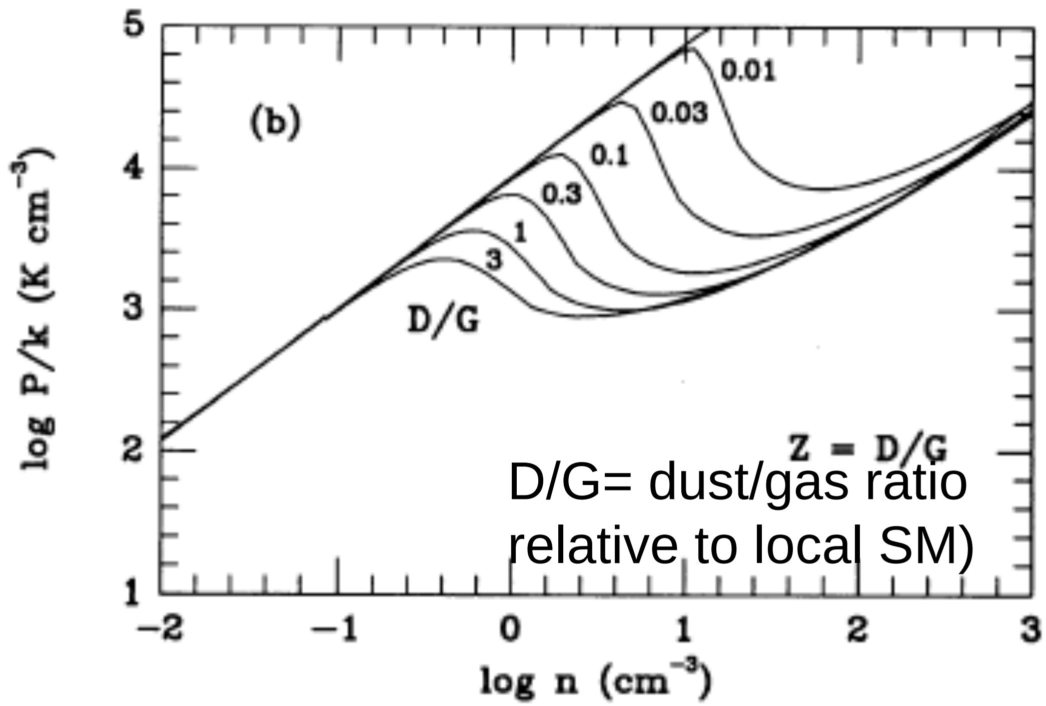
$n\lambda$ and γ *vs.* n

dashed lines: heating
solid lines: cooling

Dominant heating: **PAH** ***photoelectric effect.***
Dominant cooling: ***Fine structure lines for CNM***
H ***and forbidden lines for WNM***

Varying the Metal Abundance

- Heating & cooling of CNM is dominated by processes dependent on the heavy element abundance Z
- The dust to gas ratio is used here for Z
- Increasing *Z* reduces the range of allowed stable pressures



For the standard choice D/G =1 (local value), the allowed pressure range is quite narrow:

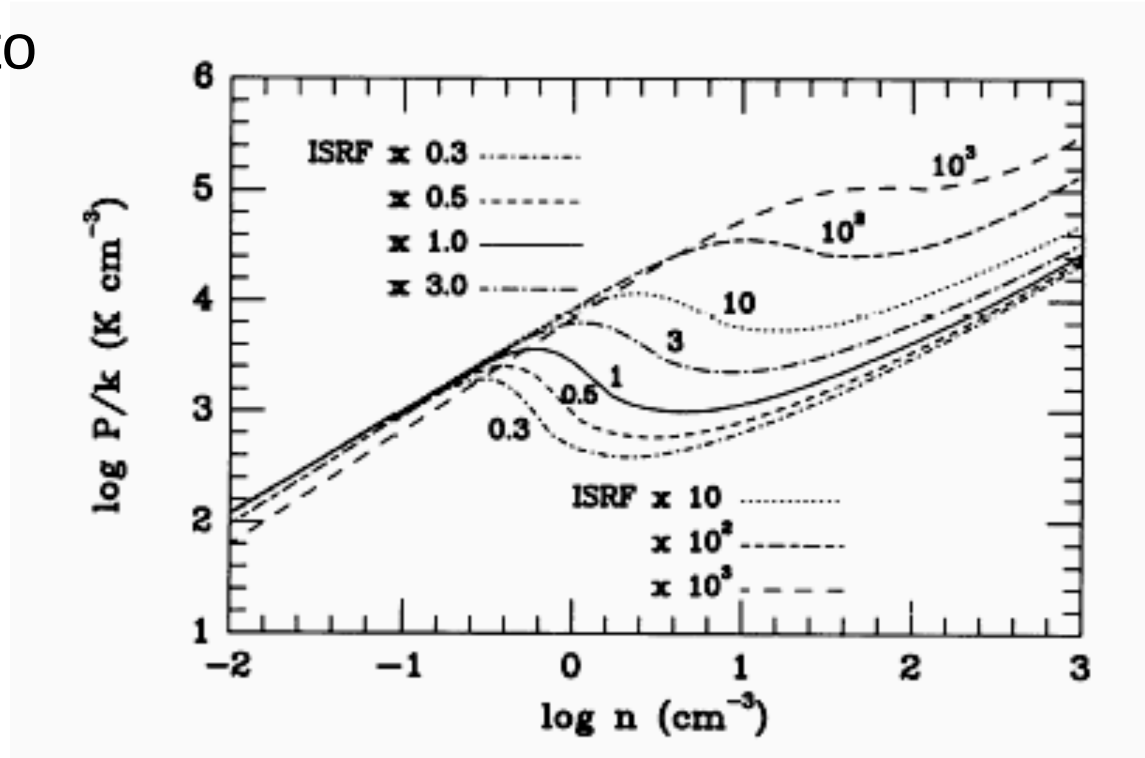
$$p/k_B = 1000\text{-}3000 \text{ cm}^{-3} \text{ K}$$

Varying the FUV Field

UV powers CNM & WNM but *T* responds weakly to small FUV changes.

Photoelectric heating is quenched by PAH and grain charging.

Large FUV increases wipe out instability, but lead to over-pressured regions.



See Wolfire et al. 1995 for other variations and Wolfire et al. 2003 for changes with galactic radius.

Summary of the Two-Phase Model

The brief for the two-phase model is laid out in detail by Wolfire et al. (1995 & 2003). The models are complex (read these papers yourselves), and the observations are limited and difficult to interpret.

Our first concern has to be whether steady state theory applies.

1. A clear observational warning comes from Heiles & Troland who find that ½ of the warm HI is in the unstable part of the phase diagram.

2. The long recombination and dynamical timescales mediate against steady state.

Theory of McKee & Ostriker

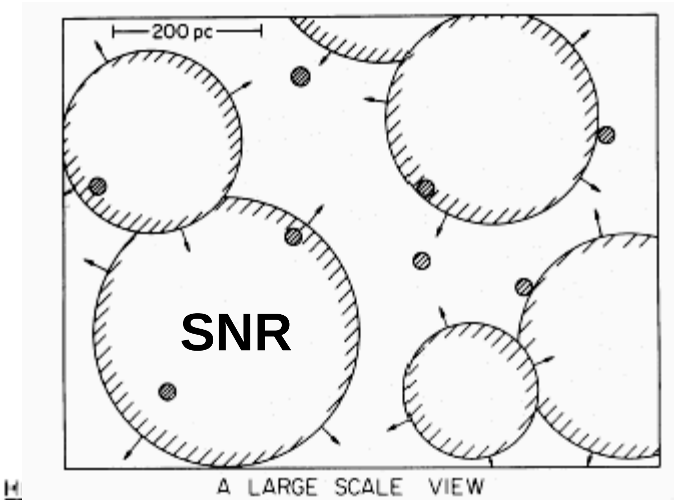
ApJ 218 148 1977

The observations indicate at least four phases (we have not discussed the HIM yet), even without including the dense midplane gas.

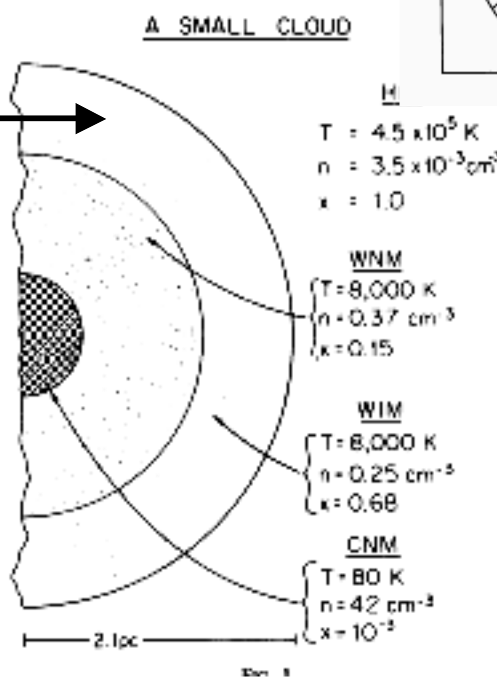
McKee & Ostriker were the first to conceive of a theory of the ISM which recognized the existence of phases other than the CNM & WNM and also gave a central role to ***supernovae*** as the primary source of mechanical energy into the ISM. Their ideas are the background for the summary of the diffuse ISM next lecture.

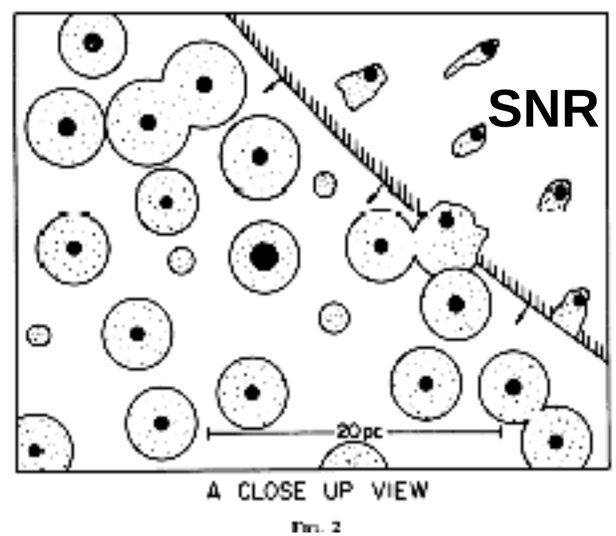
McKee-Ostriker Picture

1. 600x800 pc scale



WIM is not confined to the NM/HIM inter-face, nor does the CNM consist of compact clouds.





3. 2 pc

2. 30x40 pc