7A Notes: Angular Size/Distance and Areas

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Angular Size/Distance

As it has been alluded to in lecture, it is REALLY hard to measure distances in astronomy! What we can measure with a bit more ease is *angular* distances and separations on the sky.

Imagine a right triangle: the horizontal leg is the distance to the object d, and the vertical leg is the physical length of the object r. In astronomy, generally $r << d$ (i.e. the distance to the object is much bigger than the size of the object), so the angle subtended by the object in the sky is

$$\theta = \frac{r}{d}.$$

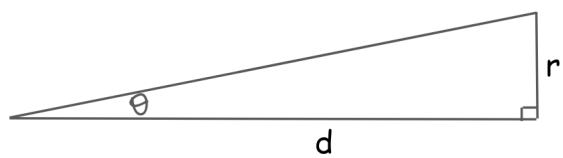

Figure 1: Angular size. You would say the angle subtended by r is θ.

A few notes about this to think about:

1. Two things can have the same *angular* size, but be different *physical* sizes! A galaxy with a physical radius of R kpc (kiloparsecs = 10^3 pc) a distance D Mpc (megaparsecs = 10^6 pc) away will have the same angular size as a different galaxy with physical radius $2R$ kpc a distance $2D$ Mpc away.

2. If you want to calculate the *physical* size of an object, you'll need to know both it's *angular* size and how far away the object is.

3. Things that are separated by a small angular distance on the sky can be separated by enormous physical distances. (They might be at almost the same position on the sky, but one could be a radial distance much farther away.)

Angular Area

You are familiar with physical areas. For example, if your room is 12 feet by 10 feet, you would say the area of your room is 120 square feet. We can also talk about angular area! It works exactly the same way: you just square the unit measuring length. So in our case, angular size is measured in square degrees, square arcseconds, etc. The SI unit for angular size is square radians, which are called *steradians*. Similarly to how an *angle* subtends a *distance*, an *angular area* subtends an *area*.

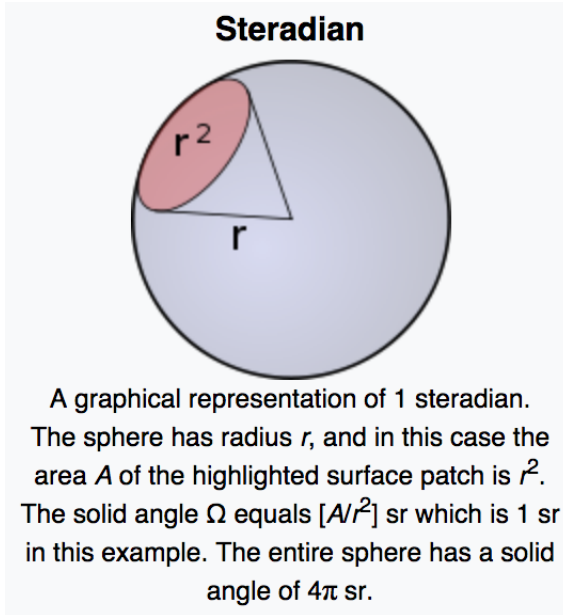

Figure 2: From Wikipedia page on Steradians.

Practice Questions

1. Q: The angular area of a sphere is 4π steradians. What is the angular area of a sphere, in square degrees?

   A: Unit conversions! Remember π radian = 180 degrees, so $\frac{180\,\mathrm{deg}}{\pi\,\mathrm{rad}} = 1$. So,

$$4\pi\,\mathrm{sr} = 4\pi\,\mathrm{rad}^2 = 4\pi\,\mathrm{rad}^2 \cdot \left(\frac{180\,\mathrm{deg}}{\pi\,\mathrm{rad}}\right)^2 \approx 41,253\,\mathrm{deg}^2.$$

2. Q: Why do we have solar eclipses?

   A: The sun and moon are both circles with roughly the same angular size on the sky! It just happens that

$$\frac{R_\odot}{D_\odot} = \frac{R_{moon}}{D_{moon}}$$