Astro/Phys 202: Astrophysical Fluid Dynamics September 11 2024

C.–P. Ma

Problem Set 1

Due: 6pm Friday September 20 on gradescope

Homework Policy: **No late homework**. You should make all efforts to solve the problems entirely on your own. You may consult a classmate for inspiration if you are stuck, but once you have received the hints, you must re-solve and write up the solutions by yourself.

0. **Reading**: Chapter 13.1 to 13.6 of Thorne & Blandford

1. **Hydrostatic Equilibrium: Troposphere**

The troposphere of the atmosphere (from sea level to about 12 km) follows approximately the equation of state for an adiabatic ideal gas.

(a) Show that the temperature gradient of the troposphere, dT/dz, is a constant that depends only on fundamental constants.

(b) Compute the values of dT/dz (in Kelvin/km) for (i) a diatomic gas, and (ii) for an isothermal gas. Compare your values with the actual gradient (called "lapse rate") of ~ 6 Kelvin/km.

(c) Find the vertical density profile $\rho(z)$ and pressure profile $P(z)$ of the troposphere. Your answers should be written as a function of z, a scale height z_s, and the density (or pressure) at sea level. At the cruising altitude of a typical commercial airliner, how are the atmospheric density and pressure compared with those at the sea level?

2. **Hydrostatic Equilibrium: Stars**

The structure of a star in hydrostatic equilibrium has a mass profile

$$M(r) = \frac{M_0}{[1 + (r_0/r)^2]^{3/2}} \tag{1}$$

where $M(r)$ is the mass enclosed within radius r, and M_0 and r_0 are constants.

(a) Calculate the density profile $\rho(r)$ of the star.

(b) Calculate the pressure profile $P(r)$ of the star.

(c) Show that the star can be described as a polytrope $P \propto \rho^{1+1/n}$, and determine the polytropic index n.

3. **Ram Pressure Stripping in Galaxy Clusters**

A galaxy cluster typically consists of hundreds to thousands of member galaxies, hot gas, and dark matter. For this problem, assume the cluster gas is spherically symmetric and is in hydrostatic equilibrium. Also assume the dark matter and gas follow the same density profile.

(a) If the density profile of the gas is a power-law: $\rho(r) \propto r^\beta$, show that the (thermal) pressure is also a power-law: $P(r) \propto r^\delta$, and relate δ to β. For an isothermal gas, what are the values of β and δ?

Assume a member galaxy at distance r from the cluster center is orbiting at the circular velocity $v_{\rm circ}(r)$. This galaxy experiences both thermal pressure $P_{\rm ther}$ from the intracluster gas and ram pressure $P_{\rm ram} = \rho v_{\rm circ}^2$ due to its motion in the cluster. Let α be the ratio of the two pressures

$$\alpha \equiv \frac{P_{\rm ther}}{P_{\rm ram}} . \qquad (2)$$

(b) Show that the (thermal) pressure gradient, $d \ln P_{\rm ther}/d \ln r$, is related to α by a simple equation. The inner, middle, and outer part of a cluster has a density profile that is approximately a power law with $\beta = -1, -2$, and -3, respectively. Using your answer to (a), compute the corresponding value of α for each of these three regions. Comment on the relative importance of thermal vs ram pressure in a cluster on its member galaxies.